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by

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in

12th International Conference on COMmunication Systems NETworkS

Bengaluru, India

Report No: IIIT/TR/2020/-1

Centre for Communications
International Institute of Information Technology
Hyderabad - 500 032, INDIA
January 2020

Copula-Based Cooperative Sensing of OFDM Signals in Cognitive Radios

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***Abstract*—This paper proposes the use of copula theory for cooperative spectrum sensing (CSS) of orthogonal frequency-division multiplexing (OFDM) based primary user (PU). A distributed detection model is assumed where secondary users (SUs) employ autocorrelation detectors (ADs) for the detection of a PU. In the presence of a PU, it is assumed that the observations across different SUs and subsequently the decision statistics are dependent. For the fusion of these dependent statistics, different copulas such as *t*-copula, Gaussian, Clayton and Gumbel are employed. In the presence of dependence among decision statistics, significant improvement in detection performance is observed while using copula theory instead of the traditional assumption of independence. Simulation results are presented to show the superiority of copula-based spectrum sensing.**

***Index Terms*—Autocorrelation detector, cognitive radio, cooperative spectrum sensing, copula, dependence**

I. Introduction

Spectrum sensing is an important component of cognitive radio (CR) networks as it provides spectrum awareness required for cognitive processing. In the CR literature, several schemes have been proposed for cooperative spectrum sensing (CSS) based on different features such as energy, autocorrelation, cyclostationarity, eigenvalues, etc., [1]–[4]. Most of this work in the literature is based on the assumption of conditional independence of observations across secondary users (SUs) conditioned on the presence/absence of a primary user (PU). The independence assumption simplifies the analysis as the joint probability density function (pdf), which is required for the log-likelihood ratio test (LLRT), can be described as the product of the marginal pdfs. However, this assumption of independence may not hold in practical scenarios. For example, if there is a strong line of sight component between an active PU and SUs, the observations will be highly dependent. In such a scenario, finding the joint pdf turns out to be a cumbersome task even for one-bit hard-decision statistics [5], [6]. Motivated by this, the present paper investigates the use of copula-theory based fusion rule for spectrum sensing where the soft-decision statistics are highly dependent.

A copula is a multivariate probability distribution for which the marginal distribution of each random variable is uniform [7]. Copulas are used to model the dependence between several random variables. In the literature, several copulas have been proposed that model different dependence structures [8]. In this paper, the following widely used copulas have been considered: Gaussian, *t*-copula and copulas belonging to the Archimedean family. In recent years, copula theory has found wide applications in finance and economics [9], [10]. Copula theory has also been used in distributed detection applications: detection of nuclear radioactive sources [11], biometrics-based automatic person recognition systems [12], classifier fusion [13] and footstep detection [14], etc. However, copula theory has not been applied to date to the problem of spectrum sensing in the CR context, which is the focus of this paper.

The specific contributions of this paper are:

- The use of copula theory is proposed for distributed detection of widely adopted orthogonal frequency-division multiplexing (OFDM) signals without making the assumption of independence among the sensor observations or decision statistics across SUs in the presence of a PU.
- The copula-based fusion rule is derived at the fusion center (FC) for the soft-decision statistics received from the SUs under the Neyman-Pearson (NP) detection criterion, which maximizes the probability of detection (P_d) under a constraint on the false alarm probability (P_{fa}).
- Performance comparison is carried out for different multivariate copulas such as Gaussian, *t*-copula and Archimedean in terms of receiver operating characteristics (ROC) for the case when the signal-to-noise ratio (SNR) values are same at all the SUs. The performance comparison is also carried out for the case when SNR values are different at different SUs.
- To confirm the relative performance of each of the considered copulas, Akaike information criterion (AIC) [15], which is generally used for choosing the best fitting copula [14], [16], is also calculated for each of the considered copulas using the same data.

The paper is organized as follows. In Section II, the system model is presented along with details of the autocorrelation detector (AD) considered in this paper. In Section III, a brief overview of copula theory and different copulas is given and the copula-based fusion rule is derived. Section IV presents the simulations results and Section V concludes the paper.

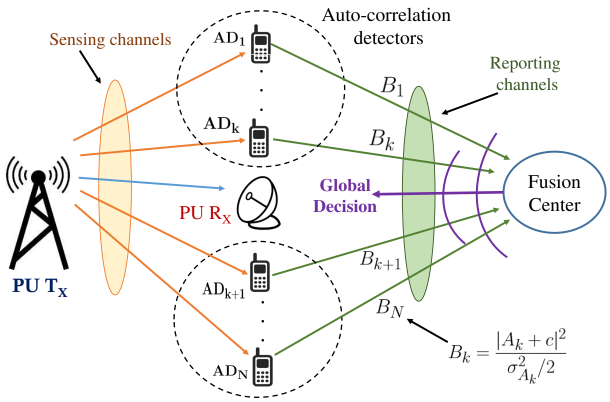

Fig. 1. System model for the considered CSS using N autocorrelation detectors (ADs).

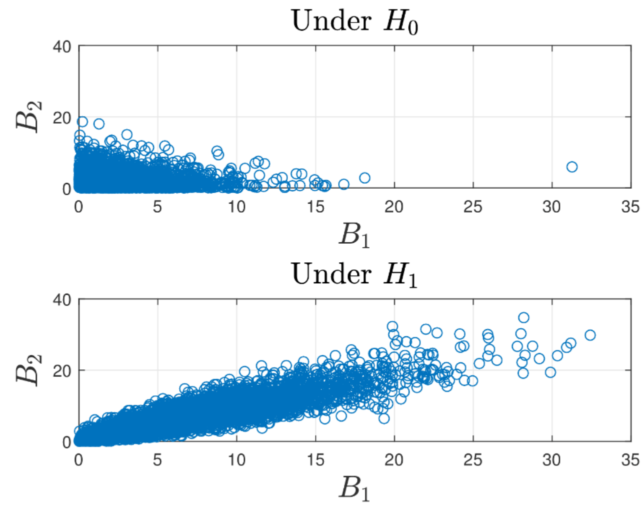

Fig. 2. Relation between auto-correlation estimates under $\mathcal{H}_0$ and $\mathcal{H}_1$ at SNR = 5 dB when $N = 2$.

II. System Model

As shown in Fig. 1, the system model for the considered CSS scenario consists of an OFDM based PU, N SUs and a FC. The sensing channels are modeled as AWGN channels. The SUs employing ADs send their respective test statistics to the FC through reporting channels which are assumed to be error-free. At the FC, a global decision is made with the help of a suitable fusion rule, which is then relayed back to all the SUs. Based on the relayed decision, the SUs can access the channel if the PU signal has been declared absent.

In spectrum sensing, the presence or absence of a PU based on locally observed signal samples can be formulated as a binary hypothesis testing problem. There are two hypotheses: $\mathcal{H}_0$, which denotes the absence of the PU signal and $\mathcal{H}_1$, which denotes the presence of the PU signal. The considered signal model is

$$\begin{aligned} \mathcal{H}_0 &: x_n[m] = w_n[m] \\ \mathcal{H}_1 &: x_n[m] = s[m] + w_n[m] \end{aligned} \tag{1}$$

for $m = 1, 2, \ldots, M_1$ and $n = 1, 2, \ldots, N$. Here, $x_n[m]$, $w_n[m]$, and $s[m]$ are the samples of the received signal, AWGN and OFDM-based PU signal, respectively at the n^{th} SU, where M_1 is the number of received signal samples at each SU. Note that the local observations of SUs, conditioned on $\mathcal{H}_1$ hypothesis are "not" assumed to be independent of each other. The noise samples $w_n[m]$ are assumed to be independent from sensor to sensor and are modelled as a complex circularly symmetric Gaussian random variables (CCSGRV) with zero mean and σ_w^2 variance. The SNR is assumed to be the same for all the detectors and σ_w^2 is assumed to be known.

A. Autocorrelation Detector (AD)

In this paper, the PU signal is assumed to be an OFDM signal, in which a cyclic prefix of T_c symbols is added in front of the data block of T_d symbols to create one OFDM symbol. Due to the presence of the cyclic prefix in an OFDM symbol, the autocorrelation value for a lag T_d is not zero, unlike AWGN whose autocorrelation value is zero for any non-zero lag value.

The AD employed in this paper is taken from [17] such that the autocorrelation estimate A_n from $M_1 = M + T_d$ received signal samples at the n^{th} SU is given by

$$A_n = \frac{1}{M} \sum_{m=1}^{M} x_n[m] x_n^*[m + T_d],$$

where '*' stands for conjugate. The n^{th} SU (or AD) then sends the soft-decision test statistic B_n to the FC as given in [17] by

$$B_n = \frac{|A_n + c|^2}{\sigma^2_{A_n}/2}. \tag{2}$$

In (2), $\sigma^2_{A_n}$ is given under the two hypotheses as

$$\begin{aligned} \mathcal{H}_0 &: \sigma^2_{A_n} = \frac{1}{M}\sigma_w^4, \\ \mathcal{H}_1 &: \sigma^2_{A_n} = \frac{1}{M}[(\sigma_s^2 + \sigma_w^2)^2 + 2\alpha^2\sigma_s^4], \end{aligned}$$

where σ_s^2 is the variance of $s[m]$ while

$$c = \frac{\alpha\sigma_w^2}{(1 + 2\alpha^2) \cdot \text{SNR} + 2} \quad \text{and} \quad \alpha = \frac{T_c}{T_d + T_c}.$$

The distribution of B_n is given in [17] by

$$\begin{aligned} \mathcal{H}_0 &: B_n \sim \chi\left(2, \frac{2Mc^2}{\sigma_w^4}\right), \\ \mathcal{H}_1 &: B_n \sim \chi\left(2, \frac{2M(\alpha\sigma_s^2 + c)^2}{(\sigma_s^2 + \sigma_w^2)^2 + 2\alpha^2\sigma_s^4}\right). \end{aligned} \tag{3}$$

Here, $\chi(k,\lambda)$ represents a non-central chi-square distribution with k degrees of freedom and non-centrality parameter λ.

B. Independence-based Fusion Rule

Let $\mathbf{B} = [B_1, B_2, \ldots, B_N]$ denote the vector of test statistics received at the FC from N ADs. The LLRT is given in [18] as

$$L(\mathbf{B}) = \log \frac{p(\mathbf{B}; \mathcal{H}_1)}{p(\mathbf{B}; \mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta. \quad (4)$$

Under the independence assumption, the joint pdfs under $\mathcal{H}_1$ and $\mathcal{H}_0$ in (4) can be written as the product of marginals to give the independence-based fusion rule as

$$L_i(\mathbf{B}) = \log \left(\prod_{n=1}^{N} \frac{p(B_n; \mathcal{H}_1)}{p(B_n; \mathcal{H}_0)} \right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_i,$$

where $p(B_n; \mathcal{H}_1)$ and $p(B_n; \mathcal{H}_0)$ are the marginal pdfs of B_n under $\mathcal{H}_1$ and $\mathcal{H}_0$ respectively.

As illustrated in Fig. 2 for a certain case, test statistics can be assumed to be independent under $\mathcal{H}_0$ but not under $\mathcal{H}_1$. So, it is expected that a different fusion rule that does not operate under the assumption of independence will perform better as it will represent the actual situation more accurately. In this paper, we employ copula theory to obtain the fusion rule.

The dependence between the test statistics in this paper is modelled using Kendall's tau (τ), which is a non-parametric rank-based measure of dependence, defined in [8] as

$$\tau = \frac{n_c - n_d}{n_c + n_d},$$

where n_c and n_d are the numbers of concordant pairs and discordant pairs respectively. For a given pair (a_i, b_i) and (a_j, b_j), let us define $z = (a_i - a_j)(b_i - b_j)$. This pair is concordant if $z > 0$ and discordant if $z < 0$. Note that the widely used Pearson's correlation coefficient, which handles only linear dependence, can not be used as the dependence between the test statistics can be non-linear [14].

III. Copula-based Cooperative Sensing

In this section, a quick overview is first given on the basics of copula theory followed by the proposed copula-based CSS. Later, the AIC criterion for copula selection is presented.

A. Basics of Copula Theory

Copula is defined as a cumulative distribution function (cdf) with uniform marginals. It helps in representing multivariate distribution functions in terms of marginal distribution functions. The role of copula in finding such a representation is described by the following theorem [10]:

Theorem 3.1 (Sklar's Theorem): Consider an N-dimensional continuous cdf F with continuous marginal cdfs $F_1, F_2, \ldots, F_N$. There exists a unique copula C such that for all y_n in $[-\infty, \infty]$

$$F(y_1, y_2, \ldots, y_N) = C\left(F_1(y_1), F_2(y_2), \ldots, F_N(y_N)\right). \quad (5)$$

On the other hand, consider a copula C and marginal cdfs $F_1, F_2, \ldots, F_N$. Then F, as defined in (5), is a multivariate cdf with marginals $F_1, F_2, \ldots, F_N$.

TABLE I
Copula densities for different copulas considered in this paper

Copulas	**Copula density** $c(\mathbf{u}) = c(u_1, u_2, \ldots, u_N)$
t-copula	$\frac{1}{\lvert\mathbf{\Sigma}\rvert^{1/2}} \frac{\Gamma\left(\frac{\nu+N}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left[\frac{\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)}\right]^N \times \frac{\left(1+\frac{\mathbf{y^T\Sigma^{-1}y}}{\nu}\right)^{-(\nu+N)/2}}{\prod_{i=1}^N \left(1+\frac{y_i^2}{\nu}\right)^{-(\nu+1)/2}}$
Gaussian	$\frac{1}{\lvert\mathbf{\Sigma}\rvert^{1/2}} \exp\left[\frac{-\mathbf{y}^T(\mathbf{\Sigma^{-1}} - \mathbf{I})\mathbf{y}}{2}\right]$
Clayton	$\left(\max\left(\sum_{i=1}^N u_i^{-\alpha} - 1, 0\right)\right)^{-(1/\alpha)}$
Gumbel	$\exp\left[-\left(\sum_{i=1}^N \left(-\ln u_i\right)^{\theta}\right)^{(1/\theta)}\right]$
Independence	1

Assuming that the copula C is differentiable, differentiating both sides of (5), we obtain the multivariate pdf

$$f(\mathbf{y}) = \left(\prod_{n=1}^{N} f(y_n)\right) c\left(F_1(y_1), \ldots, F_N(y_N)\right), \quad (6)$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_N)$ and $c(.)$ is known as the copula density given in [10] by

$$c(\mathbf{u}) = \frac{\partial^N \left(C(u_1, \ldots, u_N)\right)}{\partial u_1 \ldots \partial u_N},$$

where $\mathbf{u}$ is a vector of uniform random variables (RVs) u_i such that $u_i = F_i(y_i)$. Using (6), the joint density function can be constructed given the marginal densities and by incorporating dependency through copula density.

There are several copula functions that have been proposed in the literature and consequently, there are several possibilities of forming a joint pdf based on the choice of copula. Table I provides the copula densities for different copulas [8] used in this paper to model the joint distribution of the test statistics under the $\mathcal{H}_1$ hypothesis. Here, $\mathbf{I}$ is the identity matrix. Copulas mentioned in Table I model the dependence between RVs using different parameters that are estimated from Kendall's tau τ. $\mathbf{\Sigma}$ is the correlation matrix of RVs $\mathbf{y}$ with diagonal entries being one and non-diagonal entries being the linear correlation parameter ρ, which is defined in terms of τ [7] as

$$\rho = \sin\left(\frac{\pi\tau}{2}\right). \quad (7)$$

For Clayton and Gumbel copulas, the parameters α and θ are defined in terms of τ [8] as

$$\alpha = \frac{2\tau}{1-\tau} \quad \text{and} \quad \theta = \frac{1}{1-\tau}. \quad (8)$$

The degree of freedom ν parameter in t-copula allows one to increase or decrease the amount of tail dependence of the variables. While copula theory provides the basic framework, one needs to select the most suitable copula along with its parameters for the problem at hand.

B. Copula-based Fusion Rule

In this paper, the problem of handling the dependence between test statistics is solved by replacing the independence-based fusion rule with a copula-based fusion rule. Using (6), the pdf under $\mathcal{H}_1$ in (4) can be rewritten to give the copula-based fusion rule as

$$L_c(\mathbf{B}) = \log \left(\frac{\left(\prod_{n=1}^{N} p(B_n;\mathcal{H}_1)\right) \cdot c(\mathbf{u})}{\left(\prod_{n=1}^{N} p(B_n;\mathcal{H}_0)\right)} \right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_c, \quad (9)$$

where $c(\mathbf{u})$ is the copula density under $\mathcal{H}_1$ and $\mathbf{u}$ is given by $\mathbf{u} = \left[F_1(B_1), \ldots, F_N(B_N)\right]$. Here $c(\mathbf{u})$ is one of the copula densities mentioned in Table I, F_n is the marginal non-central chi-square cdf of B_n under $\mathcal{H}_1$. Note that under $\mathcal{H}_0$, as test statistics are independent, the joint pdf will still be the product of marginal pdfs.

C. AIC for Copula Selection

In (9), the distribution under $\mathcal{H}_1$ is the product of the marginals and the copula density. As marginals are independent of the copula used, the criterion to fit the pdf under $\mathcal{H}_1$ relies on the choice of the copula. Now, given the B_n values, the copula which fits the best is to be found among several competing copulas. AIC is a measure of the relative goodness of fit of a model and has been used in this paper to decide on the best fitting copula [14], [16]. It is defined in [15] as

$$\text{AIC} = -2l(\gamma_n) + 2q, \quad (10)$$

where, q is the number of parameters that can be adjusted to make the copula fit the data and $l(\gamma_n)$ is the maximized value of the log likelihood function $l(\gamma)$ over parameters γ given by

$$\gamma_n = \arg\max_{\gamma} \sum_{k=1}^{r} c_{\gamma}(\mathbf{u}_k). \quad (11)$$

Here, r is the number of realizations. While the AIC value is computed for different copulas with the same data, the best fitting copula is the one with the least AIC value.

IV. Simulation Results

In this section, comparison of the CSS performance of different copula-based fusion rules is carried out. The inputs to the IFFT at the transmitter are chosen from a QPSK constellation. The IFFT size is chosen to be 32. Therefore, $T_d = 32$. The cyclic prefix is chosen as $T_c = T_d/4 = 8$. The number of received signal samples is chosen as $M = 80$. For the performance comparison of the copula-based fusion rules derived in (9) using different copulas, ROC curves are plotted by fixing $\sigma_s^2 = 1$ and varying σ_w^2 resulting in different values of Kendall's tau τ which is calculated empirically using $10,000$ realizations. For a particular value of P_{fa}, the threshold of the NP detector is calculated empirically using the same number of realizations. Monte-Carlo realizations considered for the AIC computations and the ROC plots is $r = 10,000$. P_d for a particular value of P_{fa} in the ROC plot is calculated by dividing the count of number of times $L_c(\mathbf{B})$ is greater than the threshold by the number of Monte-Carlo realizations r. First, the ROC curves are plotted for the case when the SNR is the same at different SUs. Next, results are presented for the case when the SNR is different at different SUs.

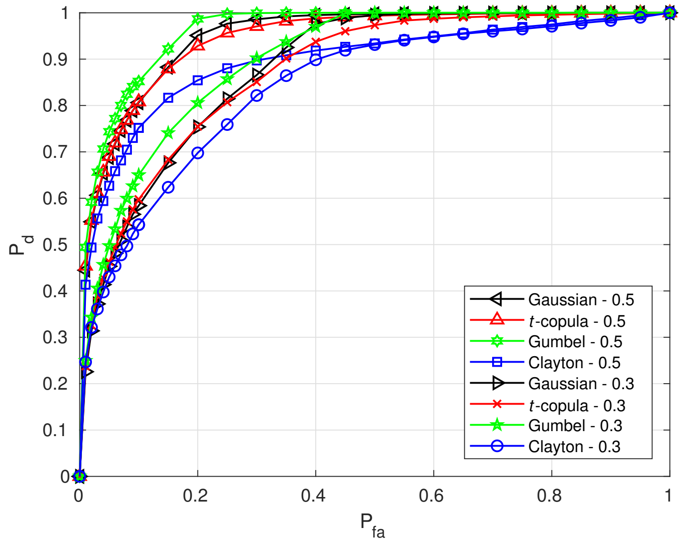

Fig. 3. ROC curves for different copulas when $N = 3$. It can be observed that the Gumbel copula performs the best followed by Gaussian, t-copula and Clayton.

A. Same SNR at different SUs

Fig. 3 compares the performance of the copula-based fusion rules for $N = 3$ and different values of τ. Here, $\nu = 6$ is used for the t-copula. For both the values of $\tau = 0.5$ (SNR $= 7$ dB) and $\tau = 0.3$ (SNR $= 3$ dB), the Gumbel copula performs the best followed by Gaussian, t-copula and Clayton. These trends in the ROC curves are in agreement with the AIC values for different copulas in Table II. The AIC value is the least for the Gumbel Copula which implies that it is the best fitting copula among the copulas considered. As such, only the Gumbel copula is considered for the rest of the simulation results among the copulas used in Fig. 3 for the case when SNR is the same at different SUs.

Fig. 4 compares the performance of the copula-based fusion rule with that of the independence-based fusion rule for $N = 3$ and different values of τ. The performances of both schemes are similar for $\tau = 0.01$ (SNR $= -8$ dB). This is expected

TABLE II
AIC values for different copulas when $N = 3$. Note that this table corresponds to Fig. 3. It can be observed that the copula for which the AIC value is the least performs the best in Fig. 3 and vice versa.

τ	**Gaussian copula**	***t*-copula**	**Gumbel copula**	**Clayton copula**
0.5	-1.32e04	-1.19e04	-1.41e04	-4.71e03
0.3	-3.65e03	-3.29e03	-4.43e03	-1.32e03

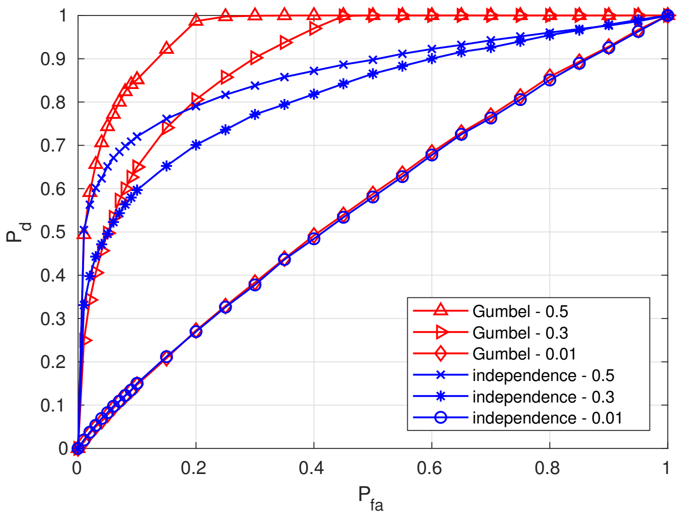

Fig. 4. ROC curves for copula-based fusion rule and independence-based fusion rule when $N = 3$. It can be observed that the copula-based fusion rule performs better than the independence-based fusion rule when the test statistics are highly dependent.

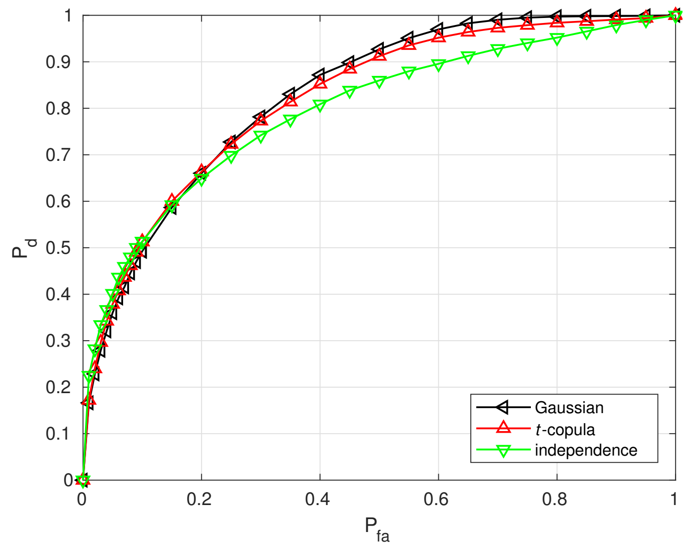

Fig. 6. ROC curves for copula-based fusion rule and independence-based fusion rule for $N = 3$ and different SNR at different SUs (-5 dB, 0 dB and 5 dB). It can be observed that the copula-based fusion rule performs better than the independence-based fusion rule.

as for weak dependence, the test statistics can be assumed to be independent. However, for the cases of higher dependence ($\tau = 0.3$ and 0.5) among the test statistics, the copula-based fusion rule outperforms the independence-based fusion rule. So, by incorporating dependence into the fusion rule and not assuming independence, CSS performance improves significantly. Moreover, the performance improvement increases with increase in dependence.

Fig. 5 compares the performance of the copula-based fusion rule with that of the independence-based fusion rule for

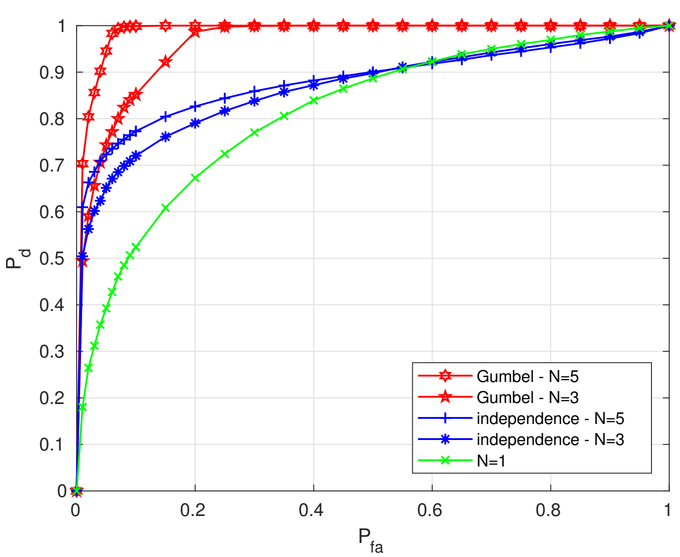

Fig. 5. ROC curves for Gumbel copula-based fusion rule and independence-based fusion rule for $\tau = 0.5$ and different values of cooperative SUs ($N = 3$ and $N = 5$). It can be observed that detection performance improves with increase in N.

$\tau = 0.5$ and different values of cooperating SUs ($N = 3$ and $N = 5$). Also as a reference, single user case $N = 1$ is also presented. It can be seen that cooperation improves the sensing performance as compared to the single-user case. The performance improvement over the single-user case increases with increase in the number of cooperating users. Even in this case, the performance of the copula-based fusion rule is much better as compared to the independence-based fusion rule. Also, the performance of the Gumbel copula-based fusion rule with $N = 3$ is better than the independence-based fusion rule with $N = 5$. It can also be seen that the single-user case performs better than the independence-based fusion rule with $N = 3$ and $N = 5$ for higher values of P_{fa}. This loss in the CSS performance of the independence-based fusion rule is caused by the use of independence assumption in the presence of heavily dependent test statistics.

B. Different SNR at different SUs

For the system model considered in Fig. 1, there can be a case when the SNR is different at different SUs. To deal with this scenario, the value of τ is calculated for each pair of SUs to capture the pairwise dependence between the SUs. For Gaussian copula and *t*-copula, linear correlation parameter ρ for every pair of SUs is calculated using (7). The correlation matrix $\boldsymbol{\Sigma}$ is then calculated with diagonal entries being one and non-diagonal entries being the different pairwise linear correlation parameters ρ's. However, in such a scenario of different SNR at different SUs, Gumbel and Clayton copulas can not be used directly as these copulas can not model the pairwise dependence. The α and θ parameters in (8) of Clayton and Gumbel copulas are not a matrix because of which the pairwise dependence captured using different pairwise τ's can not be used in these copulas.

Fig. 6 compares the performance of copula-based fusion rule with that of independence-based fusion rule for $N = 3$ and different SNRs at different SUs. The SNRs considered at different SUs are -5 dB, 0 dB and 5 dB. Values of different pairwise τ's are $\tau_{12} = 0.06$, $\tau_{13} = 0.09$ and $\tau_{23} = 0.24$. Among the copulas which can model the pairwise dependence, Gaussian copula and *t*-copula ($\nu = 6$) perform nearly the same. The copula-based fusion rule which captures the pairwise dependence performs better as compared to the independence-based fusion rule. These trends in the ROC curves are in agreement with the AIC values for different copulas in Table III.

TABLE III
AIC VALUES FOR DIFFERENT COPULAS WHEN $N = 3$. NOTE THAT THIS TABLE CORRESPONDS TO FIG. 6. IT CAN BE OBSERVED THAT THE COPULA FOR WHICH THE AIC VALUE IS THE LEAST PERFORMS THE BEST IN FIG. 6 AND VICE VERSA.

N	**Gaussian copula**	***t*-copula**
3	-1.44e03	-7.5e02

V. CONCLUSION

In this paper, a novel copula-based fusion rule was proposed for CSS of OFDM-based PU signal using dependent autocorrelation-based test statistics. By incorporating dependence into the fusion rule, significant performance improvement was observed in the sensing performance as compared to assuming independence among the SUs' test statistics. The performance improvement is seen to be a function of the dependence among the test statistics. Among different copulas compared, the Gumbel copula was shown to have the best performance followed by Gaussian, t-copula and Clayton in the scenario of same SNR at different SUs. Moreover, for the case when SNR is same at different SUs, the performance of the Gumbel copula-based fusion rule was found to be better than the independence-based fusion rule even when the number of cooperating users is less for the Gumbel copula-based fusion rule ($N = 3$) compared to the independence-based fusion rule ($N = 5$). For the case when SNR is different at different SUs, Gaussian Copula was shown to have the best performance followed by t-copula. The trends in the ROC curves are found to be in agreement with the AIC values computed for different copulas for both the scenarios.

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