A Novel Lane Merging Framework with Probabilistic Risk based Lane Selection using Time Scaled Collision Cone

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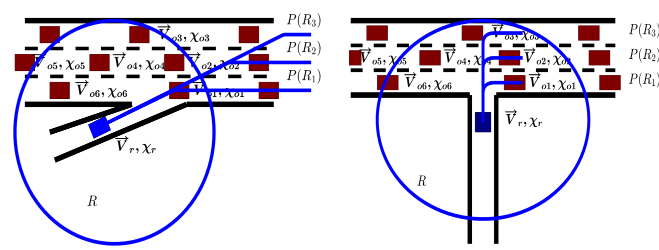
***Abstract*— Conventionally, planning frameworks for autonomous vehicles consider large safety margins and pre-defined paths for performing the merge maneuvers. These considerations often increase the wait time at the intersections leading to traffic disruption. In this paper, we present a motion planning framework for autonomous vehicles to perform merge maneuver in dense traffic. Our framework is divided into a two-layer structure, Lane Selection layer and Scale Optimization layer. The Lane Selection layer computes the likelihood of collision along the lanes. This likelihood represents the collision risk associated with each lane and is used for lane selection. Subsequently, the Scale Optimization layer solves the *time scaled collision cone* (TSCC) constraint reactively for collision-free velocities. Our framework guarantees a collision-free merging even in dense traffic with minimum disruption. Furthermore, we show the simulation results in different merging scenarios to demonstrate the efficacy of our framework.**

***Keywords*: lane merging, motion planning, time scaling, collision cone, probability, autonomous vehicles.**



(a) Highway Merging (b) T-junction Merging

Fig. 1: Common Merge Scenarios. 1a shows a general Highway Merging scenario where the autonomous vehicle (robot with position X_r and velocity $\overline{V_r}$) merges onto the highway from the tapper road. 1b shows a T-junction Merging scenario where the autonomous vehicle merges onto the highway from a T-junction. $(X_{o_n}, \overline{V_{o_n}})$ represents the positions and the velocities of the other vehicles. The blue circle represents the autonomous vehicle's perception range which ensures that there are no blind spots due to building and road infrastructure. This is achieved by integrating sensors like LiDAR and V2V (Vehicle-to-Vehicle) communication systems [1]. $P(R_i), where\ i = 1,2,3$ represents the probabilistic risk associated with each of the lanes.

I. INTRODUCTION

While driving, merging onto the traffic is one of the most difficult but frequent scenarios. Majority of accidents tend to happen during these situations due to an error in judgment that one makes while merging onto the traffic. In such cases, the driver analyzes these dynamic situations and trusts his intuition to predict the speed and the course of the oncoming vehicles for making the maneuver. Now if this problem is presented in the domain of Autonomous Driving, it becomes more convoluted. To handle such scenarios we propose a novel framework for merging with a unique lane selection technique.

The previous works on lane merging [2], [3] consider the innermost lane with large margins of safety as the feasible path. Our work extends beyond this by considering all the lanes as potential paths for merging. The principal contributions of this work includes a novel lane selection mechanism that selects the lane which has the least collision risk along its path. This is achieved by modeling the time scaling function as a probability distribution and solving for the likelihood of collision using the TSCC constraint. The collision-free velocities along the selected lane are computed by solving the TSCC constraint.

Moreover, we show that our framework reduces the wait time at the intersection significantly, leading to minimal disruption in traffic flow. Further, the framework is tested extensively in dynamic traffic conditions for the two merging scenarios as shown in Fig.1. The empirical analysis of the same is performed considering the velocities bounds, acceleration bounds, time of merging and inter-vehicle gap. It is also shown that the proposed framework is computationally efficient with run time less than $20ms$ in Section VII.

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II. RELATED WORK

There have been quite a few approaches for autonomous driving that focus on motion planning for dynamic maneuvers [4], lane changing [5], overtaking [2] and other approaches like cooperative vehicle merging [6] and vehicle merging on Automated Highway systems [7]. They span from classical trajectory planning formulations [8], [9] to modern learning-based approaches [10], [11]. However, the literature from lane merging point of view is quite sparse. [3] presents a work for lane merging, which incorporates a selection of minimal-jerk trajectory in every planning cycle, based on the cost functions for risk and comfort. The planning time is 5Hz and merging is shown in the presence of a minimal number of obstacles in a lane.

An MPC framework with velocity decomposition is presented for autonomous driving [2]. Wherein, the merging behavior is achieved by considering large safety margins. A cost function based trajectory planning with intention integrated predictions is proposed in [12]. However, this model assumes high jerk value to switch between accelerations.

In [13] an optimal control based merging behavior is realized by generating the trajectories in a semi-reactive model. But this method requires high computational power, as a large set of candidate trajectories needs to be generated to handle dynamic street scenarios. A Lyapunov-based control scheme is developed to generate the merging strategy from multiple lanes to a single lane [14].

All the approaches presented above are modeled to merge onto the innermost lane of a highway. Also, none of them present an approach to accelerate ahead of a pack of cars, provided a feasible gap is available for merging. Our approach is novel in its characterization of lane-risk and presents a unique adaptation of time scaled collision cone [2], [15] to derive a closed-form analytical solution for velocities that enable merging. A variety of maneuvers such as slowing down before a pack of vehicles or accelerating ahead of them along with merging into tight gaps, can be achieved. This is based on the inter-vehicle gap available to the autonomous vehicle while merging. In this way, the presented method contrasts from the earlier works by its ability to handle complex scenarios.

III. Overview of the Proposed Approach

To achieve a collision-free merge maneuver, our framework is divided into a two-layer structure namely lane selection layer and scale optimization layer. In the lane selection layer, first a global path is generated from the sampled way-points. Using this, a set of minimal-jerk trajectories are generated onto each lane on the highway. This is described in Section IV. The collision-risk associated with each lane is evaluated by modeling the scaling function of the TSCC constraint as a probability distribution is introduced in Section V. Then the lane with least collision risk is selected and followed.

In the scale optimization layer as described in Section VI, the TSCC constraint is posed as an optimization problem to reactively solve for the collision-free velocities along the selected path. The overall pipeline of our framework is summarized in Fig.2.

IV. Motion Planning in Merge Scenario

A. Map Generation and Global Path Planning

The proposed framework uses a map database system called OpenStreetMap (OSM) [16] and a GPS system for the localization of the autonomous vehicle on the lanes. A set of way-points are sampled from the start point to

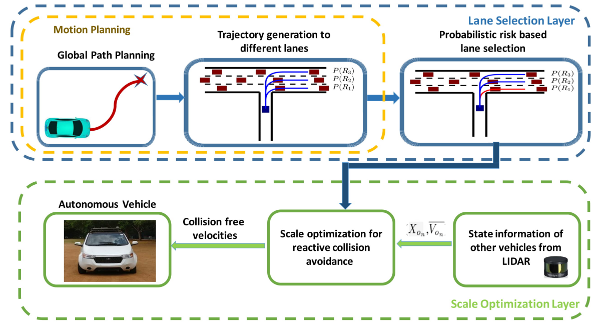

Fig. 2: Overall Pipeline of the Proposed Work

the goal point using this map to generate a global plan. This plan is generated using a 2D cubic Bézier curve which satisfies both curvature continuity and maximum curvature constraint. The control points required for the 2D Bézier curve are obtained by tailoring the formulation proposed in [8] for the merging scenario.

B. Trajectory Generation

By using the global path, a set of candidate trajectories are generated onto the available lanes. The generated trajectories are of fifth-order, in order to maintain minimal-jerk [17] along the path. A similar method is used in [9] which adapts an optimal control model for reactive autonomous navigation.

The coefficients of the quintic polynomial are obtained by considering the known initial $(x_o, \dot{x}_o, \ddot{x}_o)$ and the final state $(x_f, \dot{x}_f, \ddot{x}_f)$ and solving the following equation(1)

$$\begin{bmatrix} x_o \\ \dot{x}_o \\ \ddot{x}_o \\ x_f \\ \dot{x}_f \\ \ddot{x}_f \end{bmatrix} = \begin{bmatrix} 1 & t & t^2 & t^3 & t^4 & t^5 \\ 0 & 1 & 2t^2 & 3t^3 & 4t^3 & 5t^5 \\ 0 & 0 & 2 & 6t & 12t^2 & 20t^3 \\ 0 & 0 & 0 & 6 & 24t & 60t^2 \\ 0 & 0 & 0 & 0 & -24 & -120t \\ 0 & 0 & 0 & 0 & 0 & 120 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \tag{1}$$

where a_n, $n = \{0,1,2,3,4,5\}$ are the coefficients of the quintic polynomial.

V. Lane Selection based on Probabilistic Collision Risk

In this section, we introduce the concept of time scaling and give a brief preview of our previous work on *time scaled collision cone* (TSCC) [13], [18]. Then we present a novel lane selection mechanism by modeling the time scaling function of the TSCC constraint as a probability distribution($f_{\dot{S}}(\dot{s})$). The scaling function is characterized by its preferred scale and velocity-acceleration bounds.

A. Time Scaling

The time scaling transformation does not alter the actual path of the trajectory $\mathbf{X}(t)$, but changes the time scale from t to τ which changes the motion profiles of the autonomous vehicle, defined by the following equations

$$\dot{\boldsymbol{X}}(\tau) = \dot{\boldsymbol{X}}(t)\frac{dt}{d\tau}, \quad \ddot{\boldsymbol{X}}(\tau) = \ddot{\boldsymbol{X}}(t)(\frac{dt}{d\tau})^2 + \dot{\boldsymbol{X}}(t)\frac{d^2t}{d\tau^2} \tag{2}$$

$$\frac{dt}{d\tau} = \dot{s}, \quad \frac{d^2t}{d\tau^2} = \ddot{s} \tag{3}$$

where, $\frac{dt}{d\tau}$ is the scaling function that transforms time scale from t to τ.

We approximate $\frac{dt}{d\tau}$ as a linear function for an arbitrary time interval $[t_i \;\; t_{i+1}]$. From equation(2) we transform this arbitrary time interval to $[\tau_i \;\; \tau_{i+1}]$ in the new scale τ, given by the following relation [19].

$$\tau_{i+1} - \tau_i = \frac{2(t_{i+1} - t_i)}{\dot{s}(t_i) + \dot{s}(t_{i+1})} \tag{4}$$

The gradient of the scaling function $\frac{d^2t}{d\tau^2}$ is derived by using equation(4)

$$\frac{d^2t}{d\tau^2} \approx \frac{\dot{s}(t_{i+1})^2 - \dot{s}(t_i)^2}{2\Delta t} \tag{5}$$

Since the scaling function($\dot{s}$) is being approximated to be a monotonically linear function, $\frac{d^2t}{d\tau^2}$ is constant in the time interval $[t_i \;\; t_{i+1}]$

B. Time Scaled Collision Cone

The space of velocities that the vehicle can attain at time t_{i+1} and position $\boldsymbol{X(t_{i+1})}$ is obtained by applying the scaling transformation and is given by

$$\dot{\boldsymbol{X}}(\boldsymbol{\tau_{i+1}}) = \dot{s}(t_{i+1})\dot{\boldsymbol{X}}(\boldsymbol{t_{i+1}}) \tag{6}$$

Applying the scaling transformation(6) in the collision cone constraint [20], leads to the following time scaled variant of collision cone constraint [15].

$$\frac{((r_j)^T v_j)^2}{||v_j||^2} - ||r_j||^2 + R_j^2 < 0 \tag{7}$$

where,

$$r_j = \begin{bmatrix} x(t_{i+1}) - x_j(t_{i+1}) \\ y(t_{i+1}) - y_j(t_{i+1}) \end{bmatrix}, V_j = \begin{bmatrix} \dot{s}(t_{i+1})\dot{x}(t_{i+1}) - \dot{x}_j(t_{i+1}) \\ \dot{s}(t_{i+1})\dot{y}(t_{i+1}) - \dot{y}_j(t_{i+1}) \end{bmatrix} \tag{8}$$

In equation(8), $x(t_{i+1})$, $\dot{x}(t_{i+1})$, $y(t_{i+1})$, $\dot{y}(t_{i+1})$...etc. are known as the trajectory $\boldsymbol{X(t)}$ and scale are given. This time scaled collision cone constraint (TSCC) inequality(7) can be represented as a single variable quadratic inequality of the form

$$a_j\dot{s}(t_{i+1})^2 + b_j\dot{s}(t_{i+1}) + c_j \leq 0 \tag{9}$$

where a_j, b_j, c_j are functions of $x(t_{i+1})$, $y(t_{i+1})$, $\dot{x}(t_{i+1})$, $\dot{y}(t_{i+1})$. Solving the TSCC constraint will result in a solution space of velocities that avoids the obstacles.

C. Probabilistic Collision Risk Computation

The autonomous vehicle is collision free if it satisfies the TSCC constraint(9). By modeling the time scaling function as a probability distribution $f_{\dot{S}}(\dot{s})$ we define the collision risk as the likelihood that satisfies the TSCC constraint.

Since, the TSCC constraint(9) is a continuous function with no flat spots(having a constant value for a finite interval), the probability distribution $f_Y(y)$ associated with it can be determined by applying the concept of transformation of random variable [21].

From equation(9)

$$Y = a_j\dot{s}(t_{i+1})^2 + b_j\dot{s}(t_{i+1}) + c_j \leq 0 \tag{10}$$

The probability distribution of Y, $f_Y(y)$ is given by the following equation by using the concept of transformation of a random variable

$$f_Y(y) = \sum_{\dot{s}_i \in \S} \frac{f_{\dot{S}}(\dot{s}_i)}{|\frac{dY}{d\dot{s}}(\dot{s}_i)|} \tag{11}$$

where § is the set of all real solutions of $Y = a_j\dot{s}(t_{i+1})^2 + b_j\dot{s}(t_{i+1}) + c_j = 0$. The probability distribution of the time scaling function($\dot{s}$) is characterized as a Normal distribution (12) with the mean at the preferred time scale of the vehicle and a standard deviation based on the scale limits that the vehicle can achieve. These limits are determined by the velocity and acceleration limits of the vehicle as shown in Section VI.

$$\dot{s}(t_i) = N(\mu_i^{\dot{s}}, \sigma_i^{\dot{s}}) \tag{12}$$

In(12) $\mu_i^{\dot{s}}, \sigma_i^{\dot{s}}$ are the mean and standard deviation of the Normal distribution at the time instant t_{i+1}, which are given by

$$\mu_i^{\dot{s}} = \dot{s}_{pref}, \;\; \sigma_i^{\dot{s}} = (\dot{s}_{max} - \dot{s}_{min})/2 \tag{13}$$

$$\dot{s_1} = \frac{-b - \sqrt{b^2 - 4a(c-y)}}{2a}, \;\; \dot{s_2} = \frac{-b + \sqrt{b^2 - 4a(c-y)}}{2a} \tag{14}$$

are the roots of the equation(10). Now using these roots, the probability density function of Y can be derived by using the equation(11) as

$$f_Y(y) = \frac{f_{\dot{S}}(\dot{s_1})}{|\frac{dY}{d\dot{s}}(\dot{s_1})|} + \frac{f_{\dot{S}}(\dot{s_2})}{|\frac{dY}{d\dot{s}}(\dot{s_2})|} \tag{15}$$

Solving the equation(15),by characterizing $\dot{s}$ as $N(\mu_i^{\dot{s}}, \sigma_i^{\dot{s}})$, the probability density function of collision cone constraint(10) is derived to be (here $\mu = \mu_i^{\dot{s}}$ and $\sigma = \sigma_i^{\dot{s}}$)

$$f_Y(y) = \begin{cases} \exp\left(-\frac{2ab\mu + 2a(a\mu^2 - c + y) + b^2}{2a^2\sigma^2}\right) \frac{\left(\exp\left(\frac{(-\sqrt{K}+p)^2}{8a^2\sigma^2}\right) + \exp\left(\frac{(\sqrt{K}+p)^2}{8a^2\sigma^2}\right)\right)}{\sqrt{2\pi K\sigma^2}}, & if \quad K > 0 \\ 0, & elsewhere \end{cases} \tag{16}$$

where, $K = -4ac + 4ay + b^2$ and $p = 2a\mu + b$

From the concept of collision cone, if the autonomous vehicle is collision-free, then it should satisfy the equation(9). This leads to the following

$$P(Y < 0/O_i) \qquad i = \{1, 2, 3, ...n\} \tag{17}$$

where O_i, $i = \{1, 2, 3, ...n\}$ are the obstacles(vehicles) surrounding the autonomous vehicle.
Thus, the risk associated with each individual vehicle is computed as following.

$$P(Y < 0/O_i) = \int_{-\inf}^{0} (f_Y(y))dy \quad i = \{1, 2, 3, ...n\} \tag{18}$$

The overall risk associated with a lane ψ is then evaluated by calculating the individual risks of the vehicles, that the autonomous vehicle is probable to interact while performing the merge maneuver. The overall risk ψ_j for lane j is given by

$$\psi_j = 1 - \Big(P(Y<0/O_1)\ P(Y<0/O_2) \\ P(Y<0/O_3)......P(Y<0/O_k)\Big) \tag{19}$$

where $O_1, O_2, ...O_k$ are the vehicles that the autonomous vehicle is probable to interact while performing the merge maneuver onto a particular lane j.

This overall risk ψ_j, where $j = 1,2,3...m$ is evaluated for each lane when the vehicle enters the merge zone. Where 'm' is the number of lanes that are available for the autonomous vehicle for merging. The lane with the least risk is then selected to perform the merge maneuver. By characterizing the scaling function ($\dot{s}$) as described in equations(11),(12),(13), the risk associated with each lane is evaluated by considering the velocity and accelerations limits. As a result, the driving conditions are improved while merging onto dense traffic.

VI. Scale Optimization for Collision-free velocities

The collision-free velocities along the path $\boldsymbol{X}(t)$ are computed by solving for the scaling function($\dot{s}$) as an optimization problem. A similar approach is presented in [2].

$$\arg\min_{\dot{s}(t)} J_2 = \dot{s}(t_{i+1}) - \dot{s}_{pref} \tag{20a}$$

$$v_{min} \leq \dot{s}(t_{i+1})\sqrt{\dot{x}(t_{i+1})^2 + \dot{y}(t_{i+1})^2} \leq v_{max} \tag{20b}$$

$$\frac{a^{lon}_{min}}{\sqrt{2}} \leq \dot{s}(t_i)^2\ddot{x}(t_i) + \ddot{s}(t_i)\dot{x}(t_i) \leq \frac{a^{lon}_{max}}{\sqrt{2}} \tag{20c}$$

$$\frac{a^{lat}_{min}}{\sqrt{2}} \leq \dot{s}(t_i)^2\ddot{y}(t_i) + \ddot{s}(t_i)\dot{y}(t_i) \leq \frac{a^{lat}_{max}}{\sqrt{2}} \tag{20d}$$

$$a_j\dot{s}_i(t_{i+1}) + b_j\dot{s}(t_{i+1}) + c_j \leq 0 \forall j = 1,2,3,...,n \tag{20e}$$

Where,

$$\dot{s}_{pref} = \frac{v_{pref}}{\sqrt{\dot{x}(t_{i+1})^2 + \dot{y}(t_{i+1})^2}} \tag{21}$$

The equation(20a) is the cost function designed to compute the scaling function with minimum deviation from the preferred scale. The inequality (20b) enforces the velocity bounds and v_{min} is always non-negative taking in the fact that the autonomous vehicle should not go in reverse as it may cause disrupt the traffic flow. The inequalities (20c), (20d) enforces the longitudinal and lateral acceleration bounds respectively. The preferred scale $\dot{s}_{pref}$ is determined from the preferred velocity v_{pref}. The v_{pref} for performing the merging maneuver onto the available lanes is obtained by selecting the appropriate trajectory from the candidate trajectories generated in Section IV-B. These constraints are formulated and solved as a quadratic programming (QP) problem.

VII. EXPERIMENTAL RESULTS

A. Simulation Framework and Computational time

To evaluate the performance of the proposed framework, we simulated the two merge scenarios: T-merge and Highway merge in Gazebo [22]. The positions and velocities of the other vehicles are estimated by using a LiDAR which is mounted on the car and has range limited to $50m$. This sensor range is reasonable enough for merging and lane-changing scenarios with speeds less than $20m/s$. The speed limits of the highway vehicles are varied and the behavior of our framework is evaluated by simulating different scenarios varying from sparse to dense traffic. This variation in traffic density is achieved by varying the inter-vehicle gap(g_{iv}). The optimization problem for collision-free velocities is solved using CvxOpt [23].

The simulations are performed on Intel i7 processor @ 3.5 GHz clock speed. The average execution time for each cycle including the optimization is around 20ms and all the experimental results shown in this paper are executed at 50Hz.

B. Results

We present the results, from simulations of both T-junction merge and Highway merge to evaluate the performance of our framework. Furthermore, a through empirical analysis of the framework in common traffic scenarios with different velocities and acceleration limits is provided. Additionally, a statistical comparison of the framework with the lane selection layer and without the lane selection layer is done to demonstrate enhancement in the performance of the framework while merging.

The performance of the framework in a highway merge scenario with dense traffic is first evaluated. The velocity(V_{av}) of the autonomous vehicle before entering the merge zone is $10m/s$ and the acceleration constraints are set to $a^{lon}_{limit} = \pm 4m/s^2$ and $a^{lat}_{limit} = \pm 1m/s^2$. The minimum highway vehicle speed limit(V_{hv}) is set to $5m/s$. The snapshots of the simulation at different instances are shown in Fig.3 along with the velocity profile of the autonomous vehicle. The probability density of the collision cone (10) associated with each lane are shown in Fig.5a-5c from the innermost to the outermost lane. The collision risk evaluated by the framework is minimal for the middle lane in this scenario. Fig.3a-3c shows the merging of the autonomous vehicle onto the middle lane. The vehicle slows down smoothly to merge in between the vehicles and then maintains the speed limit of the highway to blend in the traffic flow, as shown in Fig.3d.

We then evaluate our framework in a T-junction merge scenario with the same velocities and acceleration limit settings. The snapshots of the simulation at different instances are presented in Fig.4 along with the velocity profile of the autonomous vehicle. The probability density of the collision cone equation(10) associated with each lane are shown in Fig5d-5f from the innermost lane to the outermost lane. The collision risk evaluated by the framework is minimal for the middle lane and Fig.4a-4c show the merging onto that lane. In this scenario, the autonomous vehicle accelerated smoothly to merge in between the vehicles and then slowed down smoothly to blend in the moving traffic maintaining the safe distance,

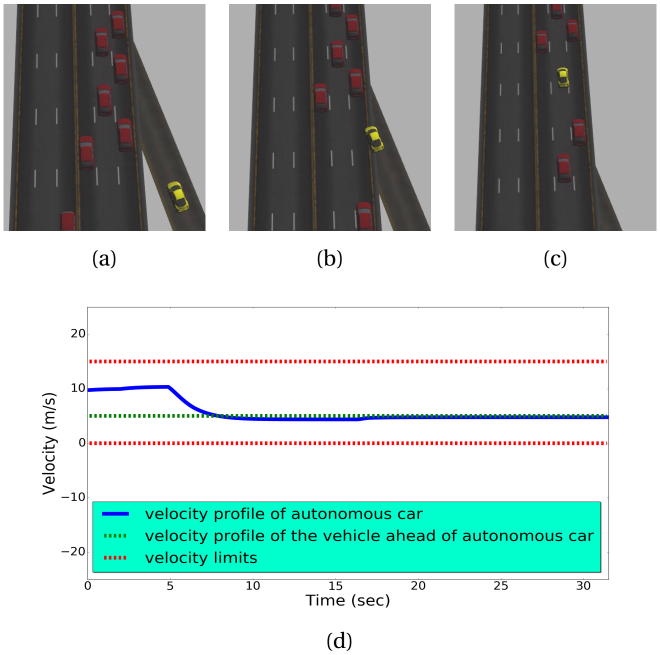

Fig. 3: Simulation for Highway Merging

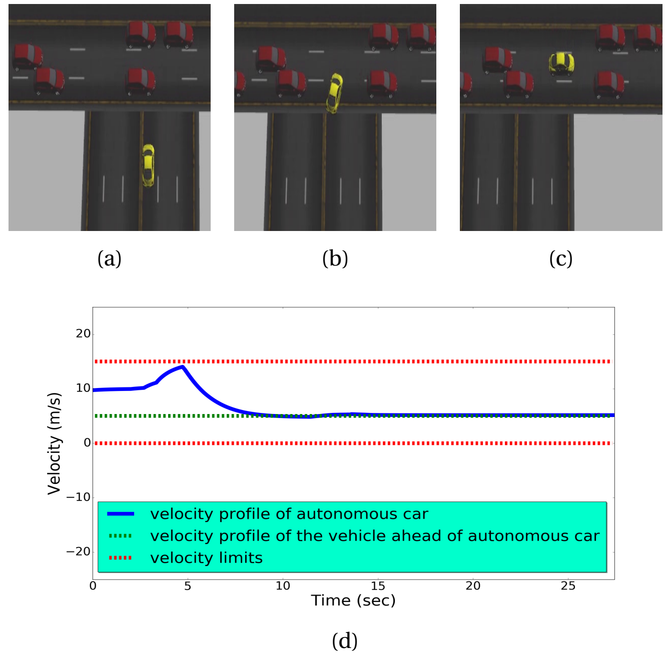

Fig. 4: Simulation for T-junction Merging

shown in Fig.4d. The simulations of the same can be found in (https://youtu.be/QJjfxyfFT5g).

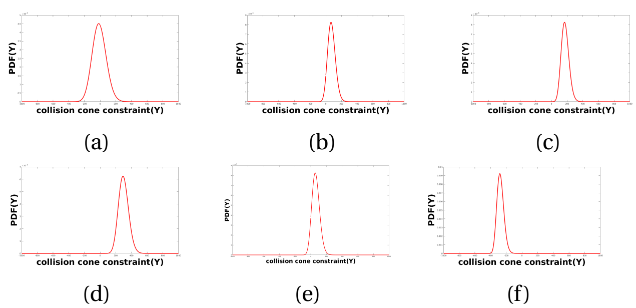

Fig. 5: Probability distributions of the time scaled collision cone constraint evaluated for each lane. Fig.5a-5c show the distributions from the innermost to the outermost lanes for the scenario shown in Fig.3 and Fig.5d-5f show the distributions from innermost lane to the outermost lanes for the scenario shown in Fig.4

1) Empirical evaluation: To get a deeper insight of the performance, we simulated our framework thoroughly in common traffic scenarios with different velocity and acceleration limits for the autonomous vehicle. The two merge cases are simulated multiple times for each parameter settings.

In TableI we present the average minimal inter-vehicle gap(g_{iv}) that the autonomous vehicle was able to merge for different highway speed limits. This scenario is simulated with the longitudinal and lateral acceleration limits of the autonomous vehicle set to $a^{lon}_{limit} = \pm 4m/s^2$ and $a^{lat}_{limit} = \pm 1m/s^2$.

TABLE I: Average minimal inter vehicle gap (g_{iv}) that the autonomous vehicle was able to merge

	T-junction merge	Highway merge
Velocity(m/s)	$g_{iv}(m)$	$g_{iv}(m)$
5	8.1	8.1
10	8.3	8.4
15	10.4	10.6
20	12.1	12.5

In Table II, the average time taken to complete the merge maneuver for different longitudinal acceleration limits is presented for both the scenarios. The results presented are obtained by simulating the scenario multiple times for each acceleration limit of the vehicle.

TABLE II: Average time taken at both the merge zones for different longitudinal acceleration limits

acc.(m/s^2)	1	2	3	4
Highway merge(sec)	15.2	12.2	8.66	6.5
T junction merge(sec)	16.1	13.2	10.7	8.2

The minimum inter-vehicle gap that the autonomous vehicle was able to merge and the time taken to complete the maneuver highlights the ability of the proposed framework to handle a dense traffic scenarios without causing any traffic bottlenecks.

2) Statistical Comparison: To analyze the lane selection method proposed in this framework a statistical comparison between the average time taken to merge with and without lane selection(LS) is done multiple times over different traffic densities and is reported in TableIII. For this the longitudinal and lateral acceleration limits of the autonomous vehicle are set to $a^{lon}_{limit} = \pm 4m/s^2$,

$a_{limit}^{lat} = \pm 1 m/s^2$ and the comparison is made for same parameter settings and traffic scenarios.

TABLE III: Qualitative analysis of the average time to merge onto different traffic scenarios with different average inter-vehicle gap(g_{iv}) for both T-junction and Highway merge with and without lane selection(LS)

Avg $g_{iv}(m)$	T-junction merge		Highway merge	
	without LS(sec)	**with LS(sec)**	**without LS(sec)**	**with LS(sec)**
25	16.3	15.1	12.7	12.1
20	17.9	16.2	16.2	15.2
15	20.6	20.1	22.1	20.4
10	27.9	25.1	21.9	21.2

These results show that the average time taken to complete the merge maneuver is significantly less with lane selection compared to without lane selection, which is prone to cause the traffic bottlenecks more frequently. Thus the proposed framework is not only better at selecting the lane having least risk of collision but also reduces the traffic disruptions.

VIII. CONCLUSION AND FUTURE WORK

In this paper, we presented a novel lane merging framework for autonomous vehicles that is capable of handling dense traffic. In contrast to previous works, we have incorporated a novel lane selection layer which would select a lane with least collision risk. This collision risk is evaluated by solving a probabilistic variant of *time scaled collision cone*(TSCC) constraint, modeled by a normally distributed scaling function. We have shown that by following the selected lane, less chaos is created in the intent displayed to the other drivers. For the collision-free velocities along the path we solved the TSCC constraint as a Scale Optimization problem. The closed form nature of this optimization problem improves the efficacy of the framework and we are able to get the update rates of about 50Hz. This aides the framework to realize realistic scenarios at high velocities in congested environments.

Our future work includes extending the proposed framework to position and velocity uncertainties of the autonomous vehicle. Along with that we explore the performance of the framework in the presence of sensor uncertainties.

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