# Data Association using Empty Convex Polygonal Regions in EKF-SLAM

Gururaj Kosuru, Satish Pedduri and K Madhava Krishna

***Abstract*— This paper proposes a new framework for data association to solve the problem of SLAM. The proposed framework has specific relevance to range scanner based EKF-SLAM. The resulting data representation enables semantic reasoning on a spatial level which reduces the misassociation of closely spaced data from different spatial configurations through the use of convex polygons to represent data from similar spatial configurations. The data representation is especially effective for association when revisiting previously mapped regions efficiently. The spatial data representation also builds an occupancy grid for the entire map.**

**We also provide a means of clustering range scan data using an adaptive threshold to be able to divide data at various ranges into clusters and dense data clustering to get more accurate data.**

## I. INTRODUCTION

A robot needs knowledge about its position within an environment to be able to navigate autonomously. A lot of work has been channelized by researchers into solving this problem as a result of which there exist different approaches based on the type of the environment. However, a single framework that can reliably function across all kinds of environment is yet to be explored. The navigation problem is divided into three parts by some researchers [1] : planning, localization and map building. This paper concentrates on the latter two which together are popularly known as Simultaneous Localization and Mapping(SLAM). The most widely used form of SLAM is the stochastic SLAM as introduced by Smith, Self and Cheeseman [3]. Stochastic SLAM relates the error in measuring the environment with the uncertainty in the robot pose which causes the landmark estimates to be dependent mutually and on the robot pose uncertainty. A practical implementation of the stochastic SLAM and the one used in this work is the Extended Kalman Filter based SLAM(EKF-SLAM) where the uncertainties and the correlations are represented with a Gaussian Probability Density Function.

All SLAM frameworks consist of a continuous component and a discrete component [2]. The continuous component is the estimation of the feature locations and the robot pose, while the discrete component is the data association problem, the problem of establishing whether two features observed during different instants of time belong to a single feature in the environment. The data association problem is preceded by the data clustering problem which determines how the features are extracted from the range scanner data. The performance of any SLAM algorithm hugely depends on the accurate modeling and association of features. Nunez et al [5] discuss a segmentation algorithm for feature modeling by extracting raw scan data into points, lines and curve segments while Fernandez [6] proposes a convolution based clustering algorithm and a hough transform based line tracker using a laser scanner. Dietmayer [7] uses a segmentation technique on a laser scan to distinguish between vehicles, persons and small objects which relies on the resolution of the scanner and a constant that can be set according to the estimated noise levels. Borges et al [13] suggest a method in which they specify the maximum distance threshold to cluster a set of data as a function of the noise in the measurement and a virtual line determined by an angle.

Diverse attempts have been made not only in the area of feature segmentation and modeling but also to improve data association. The problem of data association is usually handled using the gated nearest neighbor algorithm. Compatibility between features is estimated using the normalized innovation squared(NIS) test and the smallest Mahalanobis distance is used to select the best match.

Newman [8] first talked about a relative map filter in which the features were the distance between two point features in the map and the association was performed . Tim Bailey [9] introduced a method which takes into consideration the relative constraints and the absolute constraints based on NIS validation gate between features and makes a correspondence graph in which a maximum clique search is performed to find the largest joint compatible association set and Neira et al [10] propose a search algorithm to traverse an interpretation tree(JCBB) to find the largest number of pairings that are jointly compatible. [11], [12] perform local data association very efficiently but the revisiting problem still persists. Neira [14] introduced a sum of gaussians representation to be able to use raw scan data as features in EKF-SLAM. In this paper, we propose a method that uses the mean of a convex polygonal region formed by features to associate them respectively. Moreover, the division of space into polygons also facilitates the knowledge of occupied areas as they are not part the semantic representative structure. As an intermediate output, the occupancy grid of the environment with a semantic understanding of the surroundings is generated. The low variance of the mean of the polygon assists in improving the feature association while also making it efficient.

## II. MOTIVATION

The relative structure of an environment can play a key role in improving data association. We attempt to find a

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unique structure in each region of the map, the representative of which would be able to define the whole region. Although there has been work on the relationship between landmarks and a simple representative for a set of scan points, the spatial semantic information that defines the visibility of the robot environment is yet to be explored. Considering structured and semi-structured environments, we use the spatial structure formed between landmarks as the main criterion towards the association of features efficiently and to solve the revisiting problem. A semantic division of data prevents misassociation of features which are close metrically but separated by the topology of the environment. A compact representation of a set of features for the purpose of data association in the form of a Mean of Gaussians with a relatively low variance has been shown to perform well. Without any additional computation, this spatial representation also builds the occupancy grid of the map while performing SLAM thus leading to a metric map with occupancy information.

## III. METHODOLOGY

Our method is primarily for feature extraction and data association in EKF SLAM while also maintaining multiple visibility regions over the explored map using the point landmarks in an indoor environment consisting of a number of rooms. We explore each area of the map using a breadth first landmark search thus exploring the entire room before going to the next room and the doors between rooms are identified using a fixed unique width for doors. The robot has two laser range finders as its sensors allowing a complete 360° view. Feature extraction gathers information about the map using the observations obtained from the sensor. Data Association uses the information obtained from feature extraction to associate the current observation to previously observed features in the map.

### A. Feature Extraction

The extended kalman filter relies on parametric feature modeling. So, parametric feature extraction is essential to extract features from observations to store in the EKF state. Various parametric features can be extracted from the range scanner's observation which can be used as features in the EKF state(points,lines,etc). However, as the complexity of the feature parameterization increases, the amount of added non-linearity due to erroneous jacobian approximations also increases. We use a point feature based EKF-SLAM as it is the simplest structure for landmarks and since locations are best described by points. The process of extracting point features from the scan is done in two parts. First the data obtained from the range scanner is divided into clusters. These clusters are then divided into lines from which point features are extracted.

*1) Data Clustering:* A nearest neighbor clustering usually is given a pre-defined threshold and whenever the distance between two consecutive scan readings is greater than the threshold, the current cluster is considered complete and a new cluster is started. However, a nearest neighbor based clustering method in an indoor environment in the presence of close features can often lead to two point features from different structures to be considered to be as part of the same cluster. We propose a clustering method that clusters data on the basis of an adaptive threshold which defines the maximum distance between two points such that both might lie on the same line. For every consecutive set of readings from a single scan(say $P_1$,$P_2$), the closer of the two from the robot is first selected(say $P_1$). for the distance between $P_1$ and $P_2$ to be the minimum possible, $\angle RMP_1$ needs to be a normal angle where R is the robot position and M is the point of intersection between the line $P_1P_2$ and the perpendicular from the robot position to the line. In the case that $\angle RMP_1$ is a normal angle, the distance between the two points will be $d = 2{\cdot}r_1{\cdot}sin(\theta/2)$ where $r_1$ is the distance to the closer point from the robot. If we now start increasing the length of $P_1P_2$ by moving $P_2$ away from the robot on the line, the angle $\angle RP_1P_2$ keeps increasing from zero degrees to $90.25°$. The angle $90.25°$ is obtained by considering the isosceles triangle formed between the $R, P_1, P_2$ if the distance between $P_1P_2$ is minimal and the laser resolution is $0.5°$. By considering only feasible values, we can consider the maximum possible angle to be $85°$. By approximating the other angle to $90°$, we can set the threshold to be a maximum of $d/cos(85°)$. This is illustrated in Figure 1 where two consecutive laser hits are shown.

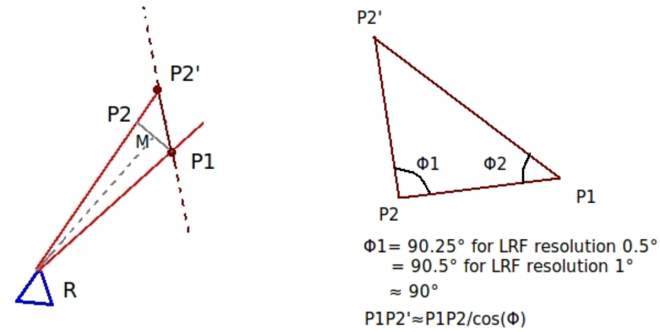


Fig. 1. Adaptive Clustering Threshold

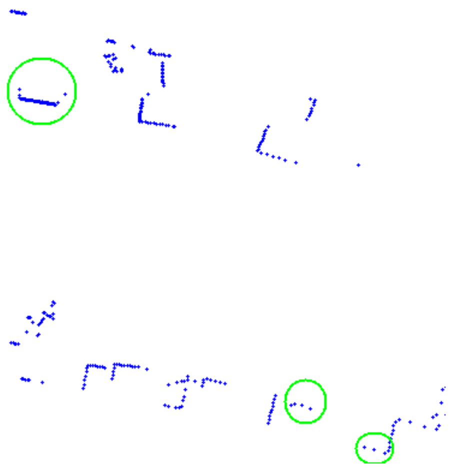

Fig. 2. Raw scan data, the laser points inside the green ellipses are sparse

Fig. 3. Raw Dense scan data, the laser points inside the green ellipses are denser

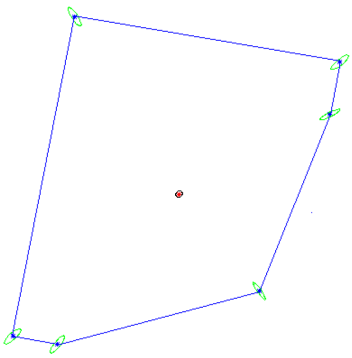

Fig. 4. Mean of Gaussians of an MECP, the green ellipses represent the variance of the vertices and the red point represents the MoG, and the ellipse around it represents its variance which is as expected, smaller than the other variances

The range scanner data represents a depth map with erroneous data. Instead of taking a laser scan for each iteration of the algorithm, the laser scanner is used to take multiple scans between two iterations. This dense depth information is used to segment data into clusters from which point feature data is extracted. We have observed that specially in the cases where the visibility of a surface is very low, a continuous dense representation improved the clustering due to the presence of a higher density of hit points. The difference in the data density is depicted in Figure 2 which is a normal laser scan and Figure 3 which is a dense laser scan.

The Ramer-Douglas-Peucker algorithm is used to divide each cluster into sets of connected line segments from which the point features are extracted from either

- the intersection of two line segments
- or the end of a line segment followed by a new cluster that does not occlude the earlier one.

*2) Polygon Division:* Point features obtained from Data Clustering are then used to find the largest(maximum cardinality) empty convex region containing the robot and bounded by the point features. We call this region the Maximum Empty Convex Polygon(MECP).

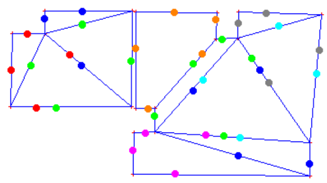

Fig. 5. MECPs extracted during the exploration of the first two rooms from Figure 7 which give a spatial semantic overview of the rooms. Adjacent edges representing an MECP have been labeled with the same color

For any convex polygon $p$ with a vertex set $v$

$$visibility(P, V) = 1 \tag{1}$$

$\forall$ points $P \in$ polygon $p$ and $\forall$ $V$ belongs to vertex set $v$. By choosing the MECP enclosing the robot, we attempt to find a unique polygon which guarantees the visibility of all its vertices as long as the robot is within the MECP. If there is more than one convex polygon with the same cardinality and enclosing the robot, both of them are considered.

The MECP is extracted from the current scan using a subroutine in Dobkin's Algorithm [15] which is an improvement over the method proposed by David Avis [16]. Given a point set $S$ of $n$ points in the euclidean plane, the algorithm finds the largest empty convex subset. This algorithm is based on a result for computing the visibility of vertices of a star shaped polygon. The basic algorithm consists of three parts which works as follows:

1) For each point $p \in S$, all other points are sorted(in counterclockwise order) by the angle subtended at $p$ which results in an ordered set $S_p$. From $S_p$, all points to the left of $p$ are removed and $p$ is added instead which results in a star-shaped polygon $P_p$. The *kernel* of $P_p$ is defined as the set of all points from which every edge of $P_p$ is visible; Obviously $p$ belongs to the kernel of $P_p$.
2) For each $p \in S$ such that the kernel of $P_p$ includes the robot position, the visibility graph $VG_p$ is computed inside $P_p$, which includes the edges of $P_p$, but does not include the visibility edges involving $p$.
3) For each $p \in S$, the largest convex chain in $VG_p$ is computed . This chain together with $p$ gives the largest convex polygon which has $p$ as the left most vertex.

These three steps are explained in detail below:

The first step requires that for every point $p \in S$, all the other points are sorted according to their angle subtended at $p$ which is done using standard sorting methods in $O(n^2)$. Similarly, removing points to the left of $p$ and constructing the star-shaped polygon $P_p$ is done in $O(n^2)$.

The second step involves the construction of a visibility graph for a star-shaped polygon of which one vertex $p$ lies on the kernel and the robot position lies inside it. Note that from our assumption that the points lie in a general position, every edge of the visibility graph will either be an edge of $P_p$ or intersect the boundary of $P_p$ in its two vertices with the edge completely inside. All the other ordered vertices are numbered as $p_1 \dots p_{n-1}$ where $n-1$ is the number of points lying to the right of $p$. The visibility graph is computed as a directed graph where every edge runs from a lower index vertex to a higher index vertex. This directed graph helps in later requirements. On visiting vertex $p_i$, all incoming edges are constructed. With each vertex $p_l$ $l \leq i$, a queue $Q_l$ is maintained which stores the starting points of some of the incoming edges of $p_l$ in the counterclockwise order. These are points $p_j$ such that $\overline{jl}$ is an edge of the visibility graph and no other point $p_k$ been reached such that $k > l$ and $\overline{jk}$ is an edge of the visibility graph. Clearly $Q_l$ is a queue that contains those points that are seen by $p_l$ but not visible after that, because $p_l$ blocks their view.

For two points $p_i$ and $p_j$, let

$$s(p_i, p_j) = \frac{Y_{p_j} - Y_{p_i}}{X_{p_j} - X_{p_i}}$$

A sqeuence of points $p_1 \ldots p_m$ define a convex chain of length m-1 if

$$s(p_1, p_2) \leq s(p_2, p_3) \leq s(p_3, p_4) \leq \ldots \leq s(p_{m-1}, p_m)$$

Step 3 calculates the longest convex vertex chain in the visibility graph $VG_p$.This is done by calculating the longest convex chains that go counterclockwise for every $p_i$ where $i = 1 \ldots n-1$. This process starts from $p_{n-1}$ and goes clockwise till it reaches $p_1$, all the while maintaining the length of the largest convex chain and the index of the vertex where it begins. Since the visibility graph was constructed as a directed graph, at every vertex $p_i$, an ordered set of incoming edges $i_1 \ldots i_{imax}$ and outgoing edges $o_1 \ldots o_{omax}$ is maintained. A clockwise scan is started from $p_{n-1}$ to $p_1$, and at every iteration, for every outgoing edge, the length of the largest convex chain that starts from that edge is stored. Every incoming edge is checked against the outgoing edges, and the lengths of those chains where it subtends a convex angle is increased by 1, or else a new convex chain of length 1 is started.

For further information, [15] may be referred to. The complexity of the entire algorithm is $O(n^2)$ where $n$ is the number of point features extracted from the scan.

### *B. Data Association*

For a global loop closing, we match data using the MECPs. The MECP vertices are just links to the indices of vertices present in the EKF framework. To match two MECPs, we use a Mahalanobis based gated matching of their centers. As each observed feature is stored as a gaussian in the state, an MECP can be compactly represented as the Mean of Gaussians(MoG) of its vertices. This representation allows a very efficient way of associating data as the variance of the resulting MoG is much lower than the variance of the individual vertices. For an MECP M, with vertices $\bar{P}_1, \overline{P_2}, \overline{P_3} \ldots \overline{P_k}$ and variance $\sigma^2_{P_1}, \sigma^2_{P_2}, \sigma^2_{P_3}, \ldots \sigma^2_{P_k}$, the gaussian for the

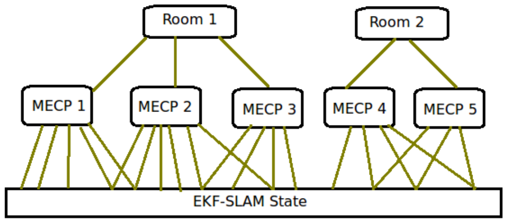


Fig. 6. The Heirarchy of data association, Data within a room can only be associated with other data from the room. Convex polygons from the same room can share vertices and edges, but Convex polygons from different rooms do not have any common vertices

center is defined as

$$\begin{aligned} g(\overline{M}, \sigma^2_M) &= \sum_{i=1}^{k} \frac{1}{k} \cdot g(\overline{P_i}, \sigma^2_{P_i}) \\ &= \frac{1}{k} \sum_{i=1}^{k} g(\overline{P_i}, \sigma^2_{P_i}) \end{aligned}$$

where $g()$ defines a gaussian function and $k$ is the number of vertices. The mean and variance $\overline{M}$ and $\sigma^2_M$ are :

$$\overline{M} = \frac{1}{k} \sum_{i=1}^{k} \overline{P_i} \quad (2)$$

$$\sigma^2_M = \left( \frac{\sum_{i=1}^{k} \sigma_{P_i}}{k} \right)^2$$

$$\sigma^2_M = \frac{1}{k^2} \left( \sum_{i=1}^{k} \sigma^2_{P_i} + \sum_{i=1}^{k} \sum_{j=1}^{k, j \neq i} \sigma_{P_i} \sigma_{P_j} \right) \quad (3)$$

where $\sigma_{P_i} \sigma_{P_j}$ is the covariance between points features $i$ and $j$. We explore each room completely before moving to the next room. This ensures that the whole room is divided into MECPs, i.e. there is no region of the room that is not occupied by atleast one polygon. During each iteration of the EKF-SLAM algorithm, once an observation is made, the features are extracted and the MECP found as described in the previous subsection.

Point features obtained in the current observation that are not part of the current MECP are associated or augmented to the EKF-SLAM state by trying to find a match with other point features in the room using a simple NIS test to find compatibility and the best match using Mahalanobis match, upon failing which the feature is added to the state as a new feature. Using the doors as a means, a room data is saved with each feature in the state. This prevents the matching of features that might appear very close metrically, but topologically are not the same.

*1) Polygon Extraction:* As each MECP's vertices are in the state vector,in order to find the MoGs of all the MECPs we first extract the MECP vertices and variances. Using the equations 2 and 3, the centers and the variances for all the MECPs are found out using the state vector for feature means and the covariance matrix for both the variance and the covariance between the features.

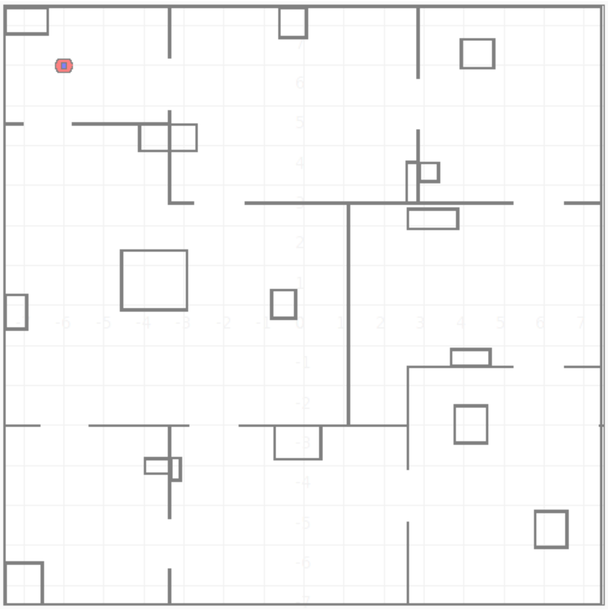

Fig. 7. SLAM environment

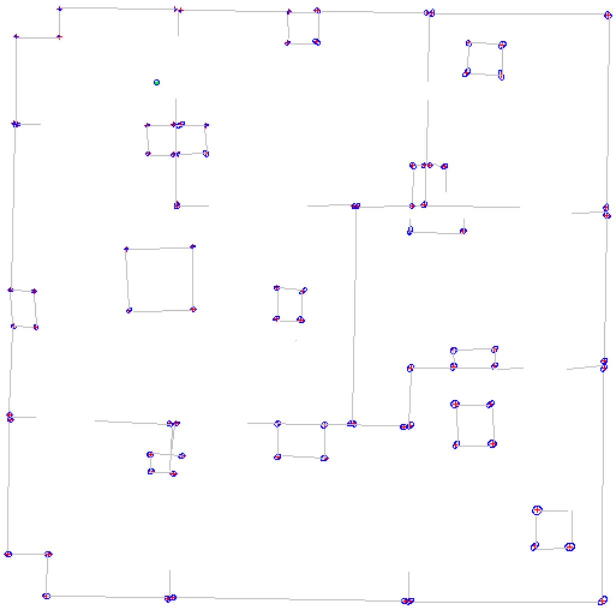

Fig. 8. EKF-SLAM results with Data association using MECPs, the lines in grey are drawn to correlate the SLAM results with the environment, the red plus symbols represent the mean of each feature and the blue ellipses around them represent their variance

*2) Polygon Matching:* Each MECP previously seen is parameterized as a gaussian. Once the MoG/center of the MECP for the current measurement is extracted, the current MoG is tested against all other former MECP MoGs to check if there is a match using the single gated NIS test and Mahalanobis match criteria and the cardinality of the MECPs. By matching the cardinality, we attempt to ensure that the MECP match is authentic. On finding a match, the respective vertices are further matched by matching two consecutive vertices using the NIS test and the Mahalanobis match, and the rest of the vertices of the polygon are matched implicitly.

When exploring a previously unexplored region, the current MECP does not match with any earlier MECPs from the state if:

- new point features are observed in the current scan although the robot is still inside one of the previous MECPs
- the robot has moved into an unexplored region.

If there is no matching MECP found, a search using the NIS test is performed across all features previously added to the state in the same room to find a match for the current observed features. The best match is found using the Mahalanobis criteria. If a match is found, then those point features are associated and the rest are augmented into the state and the current MECP is added to the MECPs.

## IV. Simulation Results

We have tested our algorithm using the Player/Stage simulator with a Pioneer P3-DX model robot and SICK laser with full field of view. We have verified the performance of our algorithm on the map in Figure 7. The results of our SLAM algorithm with Adaptive thresholding based clustering and MECP based data association is shown in Figure 8. In Figure 8, feature locations are plotted in red and the variance ellipses are plotted in blue. To show the structure of the environment and for the viewer to verify the results, we have drawn gray lines across the point features. In the current implementation, we do not consider the end points of the door as a feature. The standard deviation of the SICK laser is assumed to be $\sigma_r = 0.05$, $\sigma_\theta = 0.3°$ and that of the robot is $\sigma_{r_R} = 0.1$ and $\sigma_{\theta_R} = 1.5°$. Because of the hierarchical room based matching, it is clearly seen in the results that two very close features separated by a wall are not mismatched to one another. The MECPs for the first two rooms are shown in Figure 5.

## V. Conclusion

In this paper we have proposed a spatial representation of a set of point landmarks by a single representative point to increase the efficiency and the accuracy of data association and also build occupancy information. We have also shown a method to calculate the largest Maximum Empty Convex Polygon(MECP). For fast data association and global loop closing, we represent the polygon by a gaussian at the mean of its vertices, which is found by using a scaled sum of gaussian distribution. Using the estimated mean and variance of the MECPs from the EKF-SLAM state and variance, the currently observed MECP is checked for compatibility. As the MECPs are just stored as a set of pointers to features in the EKF-SLAM state, the MoG of all the MECPs are calculated each iteration on the fly and the EKF-SLAM state consists of the robot pose and the point features only like a generic point based EKF-SLAM framework.

## VI. Future Work

This work can be extended towards using a local submapping algorithm for each room which would reduce the variance of local features and aid in better data association in case the vertices of a scan are not matched through the MECP. The occupancy grid that is being built can be used to explore and plan the path while performing SLAM. There is a lot of scope for different spatial structures that can be used in the map which need to be explored to be able to build robust representations even in cluttered environments.

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