Quasi-Static Motion Planning on Uneven Terrain for a Wheeled Mobile Robot

Vijay Eathakota, Gattupalli Aditya and Madhava Krishna

Abstract— **In this paper we present a motion planning algorithm connecting a starting and ending goal positions of a wheeled mobile robot (WMR) with a passive variable camber (PVC) on a fully 3D uneven terrain without slipping. The overall planning framework is along the lines of the RRT (Rapidly Exploring Random Tree). The curve connecting the adjacent nodes of the RRT is a quasi-static path which is generated using the forward motion problem based on the Peshkin's minimum energy principle which combines the force and kinematic relationships of the WMR into a nonlinear optimization problem. The output of this optimization routine is a set of ordinary differential equations (ODEs) representing the non-holonomic constraints and wheel ground contact conditions of the robot along with a set of differential algebraic equations (DAEs) representing the geometric/holonomic constraints of the robot. In general a complete simulation of a WMR on a fully 3D terrain has been a difficult problem to solve. Previous methods for continuous evolution of the WMR have only incorporated the wheel ground contact constraints within the DAE framework. This work goes beyond the previous methods by incorporating the quasi-static and friction cone constraints within the DAE framework. This evolution is now extended to a motion planning algorithm which guarantees that the vehicle traverses along quasi-static stable paths.**

I. Introduction

The objective of this work is to develop a slip free motion planning algorithm for a wheeled mobile robot (WMR) having a passive variable camber (PVC) traversing on a fully 3D uneven terrain, $i.e$ starting from a known initial position of the robot platform $w.r.t$ a global frame evaluate the quasi-static stable motions enabling the robot to move to a desired final position in the 3D terrain while minimizing slip. The overall planning framework is based on the Rapidly Exploring Random Tree (RRT) algorithm.

Traditional kinematic motion planning framework based on RRT [1], Dynamic Window [2] for planar/2D terrains compute the end-effector (platform center) velocities given the joint rates. A set of ODEs are then solved to find the evolution of the platform center for the given joint rates. However for computing a completely continuous and fully 3D /$6dof$ (the position and orientation) evolution of the vehicle platform on uneven/3D terrains without slipping it is entailed to develop kinematic equations relating the joint rates to that of the velocity of the platform and the wheelground contact velocities. Such a formulation can be obtained by incorporating Montana's kinematic equations of contact[10], which is dovetailed in the quasi-static planning formulation of this paper.

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What is also important while evolving on uneven terrain is the ability of the framework to provide for the satisfaction of holonomic constraints right through the evolution, which is not of such concern in planar terrains. As rightfully pointed out in [3], the kinematic analysis on uneven terrain involves solution of a mixed holonomic-nonholonomic set of equations with the characteristics that are distinct from mobile robots on planar/even terrain. This is also achieved in this framework by incorporating the kinematic contact constraints due to Montana[10] along with a set of differential algebraic equations solved simultaneously with the ODEs. Such a framework of ODEs and DAEs have not been part of the evolutions proposed in [11,12,13] but have been considered in the works of Auchter [5] and Chakraborty and Ghosal [4]. The evolution as described in [4,5] of the WMR, assumes kinematic no slip as part of its framework by fixing the appropriate contact velocities to be zero through the Montana's kinematic equations for contact. However the vehicle can undergo dynamic slip if the friction cone constraints get violated.

In this paper we go beyond [4,5] by depicting an evolution of a WMR that also integrates quasi-static constraints into the DAE-ODE framework. Thereby the evolution provides for a complete $6dof$ evolution, where the $6dof$ pose of the vehicle platform and its rates of changes are obtained for any instant. Along with that, the evolution makes sure that the mixture of holonomic and nonholonomic constraints are satisfied along with kinematic no-slip and friction cone constraints there by providing for a robust quasi-statically stable evolution of the vehicle. This is the main contribution of this paper. The extension of the quasi-static evolution into a motion planner through the RRT paradigm is another contribution of this work.

II. Literature review

Wheeled mobile robots for navigation on uneven terrain have to be equipped with the essential degrees of freedom to negotiate the undulations on the terrain. Many such mechanisms have been reported in literature. One such mechanism was proposed by Sreenivasan and Choi [14] wherein two wheels were connected with a variable length axle (VLA) having a prismatic joint to overcome slip during terrain navigation and in [3] they developed a motion planning algorithm on uneven surface for a WMR having a variable length axle (VLA). Chakroborty and Ghosal [4] improved on this mechanism by using a passive variable camber (PVC). In [8] Cherif developed a two-level motion planning algorithm for all terrain vehicles using a physical modelling approach.

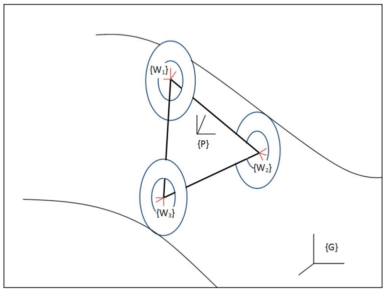

Fig. 1. WMR on uneven terrain

Amongst the previous works surveyed, to the best of our knowledge only Chakroborty and Ghosal [4], Auchter and Moore [5] (using the framework in [7]), have provided a framework for developing the slip free kinematic equations of the WMR with PVC which relate the platform velocties and the wheel-ground contact velocities to that of the joint rates. In [15] a framework for quasi-static simulation of WMR on uneven terrain is provided. In [16], a near optimal trajectory generation methodology has been proposed for mobile robots on uneven terrain however the complete $6dof$ evolution of the vehicle on uneven terrain has not been provided.

A computational simulation of a fully $6dof$ pose evolution of a vehicle platform on rough and undulating 3D terrains has proved to be a very challenging problem. Most of the previous methods have only computed an evolution on flat terrains even when the terrain is not flat and is undulating. The idea here has been to compute a kinematically consistent path such as a CC steer assuming the terrain to be flat and hope that a suitable control law would make sure that the vehicles pitch and roll are somehow adjusted to attain the yaw changes given by the CC steer curve, but are unable to predict the $6dof$ pose of the vehicle ([11],[12] and [13]). But while traversing uneven terrain, the position and the orientation (all $6dofs$) of the center of the platform of the WMR vary with the ground profile as well as any dof that the mechanism may have and hence a formulation that incorporates the holonomic constraints (DAEs),as suggested in [3], have to be used for a robust kinematic model of the WMR. To the best of our knowledge previous work that provides a complete kinematic model of the WMR with a passive variable camber (PVC) that is able to predict continuously the complete $6dof$ evolution of the vehicle platform using DAEs can be found in [4] and [5] . However these methods have not integrated the quasi static constraints and friction cone constraints into their framework, which is now possible through the framework described in this paper. The formulation used in this paper results in a set of ODEs representing the non-holonomic and holonomic constraints of the vehicle on uneven terrain which enables us to express the unactuated /dependent variables of the system in terms of the actuated/free variables of the system by the methodology provided in [17].

III. Motion Planning Framework

Let us consider a 3 wheeled mobile robot having a PVC traversing an uneven terrain as shown in the Fig.1 . As mentioned in [5], a WMR with PVC on uneven terrain is analogous to a multifingured palm (WMR) grasping an object (ground). We use this analogy in our framework to develop the motion planning algorithm.The objective of our work is to plan a quasi-static stable path that connects the given the initial and final positions of the robot on the uneven terrain. We assume that the robot has torus wheels which contact the terrain at a single point with Coloumb friction constraints. The overall planning framework is based on the RRT algorithm. The curve connecting the two adjacent nodes of the RRT is obtained using the forward object motion problem proposed by Trinkle [6] in which the kinematic constraints, the quasi-static equilibrium constraints and friction cone constraints are combined into a nonlinear optimization problem using the Peshkin's minimum energy principle which simply states that [6] "the system at each instant chooses the easiest motion while satisfying all the constraints". This principle applies to only quasi-static systems subject to forces of constraint (i.e., normal forces arising due to contacts among rigid bodies), Coloumb friction forces and forces independent of velocity. This optimization represents the instantaneous forward kinematic equations of motion of the WMR on uneven terrain whose constraiints are the non-holonomic velocity constraints of the contact points of the wheel $w.r.t$ the ground, the friction cone constraints which ensure kinematic no-slip and the static-equilibrium constraints. The input to this optimzation routine is the vector of joint rates of the robot, the current contact configuration and the effective coefficient of friction between the wheels and the ground. The solution to this optimization routine yields the contact forces and the linear and angular velocties of the robot platform $w.r.t$ the ground. The Forward motion problem for a three wheeled mobile robot on uneven terrain can thus be stated as follows

step 1 For the given joint rates of the robot and the contact parameters of the wheels with the ground at the current time step determine the velocity of the platform with respect to the ground and the corresponding contact forces which ensure quasi-static equilibrium. This can be achieved by solving the nonlinear optimization problem which will be explained in later sections.

step 2 After obtaining the platform velocities from step 1 determine the wheel ground contact velocities which will be the input to Montana's kinematic equations of contact and integrate numerically a set of ordinary differential equations (ODEs) which represent the kinematics of the robot (explained in detail in later sections) to obtain the new contact configuration.

Steps **1** and **2** are solved iteratively to obtain the forward motion of the WMR. Now for a range of joint rates of the robot the trees of the RRT are propagated by iteratively solving the steps 1 and 2 for a particular joint rate and expanding the tree which is closest to the goal point.

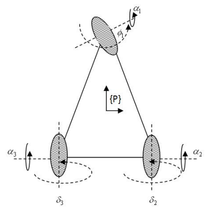

Fig. 2. WMR joints

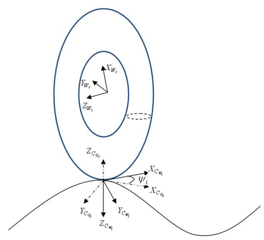

Fig. 3. torus wheel and ground frame assignments

IV. Kinematics of the WMR

Definitions : For any two reference frames $\{A\}$ and $\{B\}$, $\{R_{AB}, P_{AB}\} \in SE(3)$ is the tranformation matrix of $\{B\}$ $w.r.t$ $\{A\}$, where $R_{AB} \in SO(3)$ is the rotation matrix of the frame $\{B\}$ $w.r.t$ frame $\{A\}$ and $P_{AB} \in R^3$ is the position vector of the origin of frame $\{B\}$ $w.r.t$ $\{A\}$.
The velocity vector of the frame $\{B\}$ $w.r.t$ {A}expressed in the body frame ($V^B_{AB} \in R^6$) is given by

$$V^B_{AB} = \begin{bmatrix} \upsilon^B_{AB} \\ \\ \omega^B_{AB} \end{bmatrix} \tag{1}$$

Where $\upsilon^B_{AB} = R_{AB}\dot{P}_{AB}$ and $\omega^B_{AB} = R^T_{AB}\hat{\dot{R}}_{AB}$,$\hat{}$ operator extracts the vector associated with the skew symmetric matrix. For any three reference frames $\{A\}$, $\{B\}$ and $\{C\}$ we have

$$V^C_{AC} = Ad^{-1}_{BC}V^B_{AB} + V^C_{BC} \tag{2}$$

Where the adjoint transformation matrix and its corresponding inverse between frames $\{A\}$ and $\{B\}$ is given by

$$Ad_{AB} = \begin{bmatrix} R_{AB} & R_{AB}\hat{P_{AB}} \\ 0_{3\times3} & R_{AB} \end{bmatrix}$$

$$Ad^{-1}_{AB} = \begin{bmatrix} R^T_{AB} & -R^T_{AB}\hat{P_{AB}} \\ 0_{3\times3} & R^T_{AB} \end{bmatrix}$$

Frame Assignments: Figs. 1 and 3 show the details of the frame assignments we have considered in our analysis. $\{G\}$ is the frame assigned to the ground frame, $\{P\}$ is the frame fixed at the center of mass of the platfrom. $\{C_{G_i}\}$ is the frame fixed on the ground, at the contact point of the i^{th} wheel with the ground. $\{W_i\}$ is the frame assigned to the center of the i^{th} wheel and $\{C_{W_i}\}$ is the frame fixed on the wheel at the contact point. ψ_i is the angle between $x_{C_{G_i}}$ and $x_{C_{W_i}}$. As can be seen from Fig.2 the front wheel (W_1) is steerable, the angle of steer is given by , ϕ_1, and the rear wheels (W_2 and W_3) have a passive variable camber joint whose angles are given by ,δ_2 and δ_3 respectively.
α_i,$\forall i = \{1, 2, 3\}$ is the angle of rotation of the wheels about the $Z_{W\,i}$ axis.
Velocity relationships:We assume all the velocities expressed in the body frame unless otherwise stated. For the i^{th} kinematic chain we have.

$$V_{PG} = Ad^{-1}_{C_{G_i}G}V_{PC_{G_i}} \tag{3}$$

also we have

$$V_{PC_{W_i}} = Ad^{-1}_{W_iC_{W_i}}V_{PW_i} \tag{4}$$

Also $\{V_{PW_i}\}$ is given by

$$V_{PW_i} = J_{PW_i}(\theta_i)\dot{\theta}_i \tag{5}$$

Where J_{PW_i} is the Jacobian between the platform frame $\{P\}$ and the center wheel $\{W_i\}$ and $\dot{\theta}_i$ is the corresponding joint rate of the i^{th} kinematic chain. Also we have

$$\dot{\theta}_1 = \begin{bmatrix} \dot{\phi_1} & \dot{\alpha_1} \end{bmatrix}, \dot{\theta}_2 = \begin{bmatrix} \dot{\delta_2} & \dot{\alpha_2} \end{bmatrix}, \dot{\theta}_3 = \begin{bmatrix} \dot{\delta_3} & \dot{\alpha_3} \end{bmatrix}$$

For the frames $\{C_{W_i}\}$ and $\{C_{G_i}\}$ we have

$$A_{\psi_i} = Ad_{C_{W_i}C_{G_i}} \tag{6}$$

Where

$$Ad_{C_{W_i}C_{G_i}} = \begin{bmatrix} R_{\psi_i} & 0_{3\times3} \\ 0_{3\times3} & R_{\psi_i} \end{bmatrix}$$

and

$$R_{\psi_i} = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ -\sin\psi_i & -\cos\psi_i & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Using equation (2) for the frames $\{P\}$, $\{C_{W_i}\}$ and $\{C_{G_i}\}$ we have $V_{PC_{G_i}} = A_{\psi_i}V_{PC_{W_i}} + V_{C_{W_i}C_{G_i}}$ and can be re-written as

$$V_{PC_{G_i}} = A_{\psi_i}V_{PC_{W_i}} - V_{C_{G_i}C_{W_i}} \tag{7}$$

$V_{C_{G_i}C_{W_i}}$ represents the relative velocties of the contact frames $\{C_{W_i}\}$ $w.r.t$ $\{C_{G_i}\}$ in the respective body frames. We represent this velocity vector as

$$V_{C_{G_i}C_{W_i}} = \begin{bmatrix} v^i_x & v^i_y & v^i_z & \omega^i_x & \omega^i_y & \omega^i_z \end{bmatrix}^T$$

Using equations (3), (4) ,(5), (6) and (7) we have

$$\begin{bmatrix} v^i_x \\ v^i_y \\ v^i_z \\ \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{bmatrix} = Ad^{-1}_{W_iC_{W_i}}J_{PW_i}(\theta_i)\dot{\theta}_i - A_{\psi_i}V^i_{PG} \tag{8}$$

where $V_{PG}^{i} = Ad_{GC_{G_i}}^{-1} V_{PG}$. The velocity vector of the frame $\{P\}$ $w.r.t$ $\{G\}$ has

$$V_{PG} = \begin{bmatrix} \upsilon_{PG} \\ \\ \omega_{PG} \end{bmatrix}$$

Where υ_{PG} and ω_{PG} are the linear and angular velocity components of the V_{PG} respectively. Further simplifying (8) we have

$$\begin{aligned} \begin{bmatrix} v_x^i \\ v_y^i \\ v_z^i \end{bmatrix} &= R_{W_iC_{W_i}}^T J_{PW_iu}(\theta_i)\dot{\theta}_i \\ &\quad - R_{W_iC_{W_i}}^T \hat{P_{W_iC_{W_i}}} J_{PW_il}(\theta_i)\dot{\theta}_i \\ &\quad - R_{GC_{W_i}}^T \upsilon_{PG} + R_{GC_{W_i}}^T \hat{P_{GC_{W_i}}} \omega_{PG} \\ \begin{bmatrix} \omega_x^i \\ \omega_y^i \\ \omega_z^i \end{bmatrix} &= R_{W_iC_{W_i}}^T J_{PW_il}(\theta_i)\dot{\theta}_i - R_{GC_{W_i}}^T \omega_{PG} \end{aligned} \tag{9}$$

Where $J_{PW_iu}(\theta_i)$ and $J_{PW_il}(\theta_i)$ are the upper and lower partitions of the the Jacobian $J_{PW_i}(\theta_i)$ respectively. For pure rolling we have

$$\begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix} = 0 \tag{10}$$

The constraint that ensures that the wheel does not leave the contact with the terrain is given by

$$v_z^i = 0 \tag{11}$$

The constraints for pure sliding is given by

$$\begin{bmatrix} \omega_x^i \\ \omega_y^i \\ \omega_z^i \end{bmatrix} = 0 \tag{12}$$

A. Montana's kinematics of contact

Let $[u_{w_i}, v_{w_i}, f_w(u_{w_i}, v_{w_i})]^T$ be the parameterization of the contact point on the torus wheel with the ground and $[u_{g_i}, v_{g_i}, f_g(u_{g_i}, v_{g_i})]^T$be the parameterization of the contact point on the ground with the wheel. Also let $\{M_g, K_g, T_g\}$ and $\{M_{w_i}, K_{w_i}, T_{w_i}\}$ be the metric, curvature and the torsion forms of the ground and the i^{th} wheel respectively. Then the variation of the contact parameters $(u_{w_i}, v_{w_i}, u_{g_i}, v_{g_i}, \psi_i)$ $w.r.t$ time is given by Montana's kinematic equations of contact [10]

$$\begin{aligned} &\begin{bmatrix} \dot{u_{w_i}} \\ \dot{v_{w_i}} \end{bmatrix} = M_{w_i}^{-1} K^{-1} \left(\begin{bmatrix} -\omega_y^i \\ \omega_x^i \end{bmatrix} - K^* \begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix} \right) \\ &\begin{bmatrix} \dot{u_{g_i}} \\ \dot{v_{g_i}} \end{bmatrix} = M_g^{-1} r_{\psi_i} K^{-1} \left(\begin{bmatrix} -\omega_y^i \\ \omega_x^i \end{bmatrix} + K_{w_i} \begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix} \right) \\ &\dot{\psi_i} = \omega_z^i + T_{w_i} M_{w_i} \begin{bmatrix} \dot{u_{w_i}} \\ \dot{v_{w_i}} \end{bmatrix} + T_g M_g \begin{bmatrix} \dot{u_{g_i}} \\ \dot{v_{g_i}} \end{bmatrix} \\ &0 = v_z^i \end{aligned} \tag{13}$$

Where $K = (K_{w_i} + K^*)$ is the relative curvature matrix, $K^* = r_{\psi_i} K_g r_{\psi_i}$ and $r_{\psi_i} = \begin{bmatrix} \cos\psi_i & -\sin\psi_i \\ -\sin\psi_i & -\cos\psi_i \end{bmatrix}$ is the $2D$ representation of the frame $\{C_{W_i}\}$ $w.r.t$ $\{C_{G_i}\}$.

B. Kinematic Equations of the Robot

Using equation (9) as input to (13) we can form fifteen ordinary differential equations (ODEs) $\forall i = \{1, 2, 3\}$. Also the closed loop kinematic chains give rise to a set of constraints know as the holonomic constraints on the robot and ground parameters which can be written as.

$$\begin{aligned} \{R_{PG}, P_{PG}\}_{W_1} &- \{R_{PG}, P_{PG}\}_{W_2} = 0 \\ \{R_{PG}, P_{PG}\}_{W_1} &- \{R_{PG}, P_{PG}\}_{W_3} = 0 \end{aligned} \tag{14}$$

These set of algebraic constraints of the form $H(\Theta) = 0$ can be differentiated to a set of ODEs of the form

$$J(\Theta)\dot{\Theta} + \sigma H(\Theta) = 0 \tag{15}$$

where $J(\Theta) = \frac{\partial H}{\partial \Theta}$ and Θ is the set of wheel ground parameters and the robot joint angles. The instantaneous degrees of freedom ($d.o.f$) of the WMR can be found out to be =3 ([4],[5]) and hence only 3 of the 6 joint variables are actuated which can be chosen as $\dot{\phi_1}$, $\dot{\alpha_2}$ and $\dot{\alpha_3}$.

Hence we have

$$\dot{\theta} = \dot{\theta_d} \tag{16}$$

Where $\theta = \begin{bmatrix} \phi_1 & \alpha_1 & \delta_2 & \alpha_2 & \delta_3 & \alpha_3 \end{bmatrix}^T$ and θ_d are the desired joint rates of the robot.

The equations (13) $\forall i = \{1, 2, 3\}$, (15) and (16) form a set of ODEs which represent the unified holonomic and non-holonomic constraints of the WMR [17]. These set of equations which represent the kinematics of the WMR can be integrated numerically to get the configuration parameters of the system after each time instant.

C. Force-Moment Analysis of the WMR

The forces acting the $\{C_{W_i}\}$ frame are $f_{C_{W_i}} = [f_{xi}, f_{yi}, f_{zi}]^T$, where f_{xi}, f_{yi} are the components of the tangential forces acting at the local frame and f_{zi} is the normal force component acting from the contact point towards the center of the torus cross section. Hence the wrench basis at the $\{C_{W_i}\}$ frame is given by

$$B_{C_{W_i}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The wrench acting at the platform frame $\{P\}$ due to $f_{C_{W_i}}$ is given by

$$F_{PC_{W_i}} = G_{ri} f_{C_{W_i}} \tag{17}$$

Where

$$G_{ri} = \begin{bmatrix} R_{PC_{W_i}} & 0_{3\times3} \\ \hat{P_{PC_{W_i}}} R_{PC_{W_i}} & R_{PC_{W_i}} \end{bmatrix} B_{C_{W_i}}$$

is the grasp matrix of the i^{th} kinematic chain. Hence the total wrench acting on the platform frame $\{P\}$ due to all the contact points is given by

$$F_P = G_r f \tag{18}$$

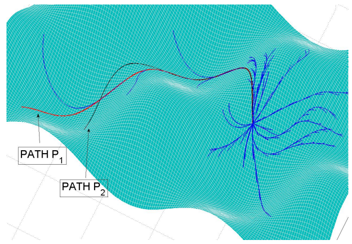

Fig. 4. RRT algorithm for different initial configurations

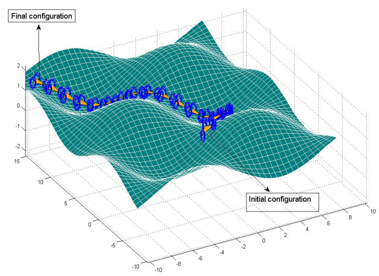

Fig. 5. Snapshots of the WMR on Uneven Terrain

where the grasp matrix $G_r \in R^{6\times 9}$ is given by

$$G_r = \left[\begin{array}{ccc} G_{r1}B_{C_{W_1}} & G_{r2}B_{C_{W_2}} & G_{r3}B_{C_{W_3}} \end{array} \right]$$

and $f = [f_{x1}, f_{y1}, f_{z1}, f_{x2}, f_{y2}, f_{z2}, f_{x3}, f_{y3}, f_{z3}]^T$ is the vector of all the contact forces at each of the contact points. For the vehicle to move without slip at the contact points of the wheels with terrain the forces acting at the contact point must satisfy the friction cone constraints which are given by

$$f_{xi}^2 + f_{yi}^2 \leq \mu^2 f_{zi}^2 \tag{19}$$

Also for the wheel to maintain contact with the ground the normal force component at the contact point should be non-negative,$i.e$,

$$f_{zi} \geq 0 \tag{20}$$

For the vehicle to be in quasi-static equilibrium the total wrench on the platform F_P should balance the total external wrench. Hence the quasi-static force-moment balance equation is given by

$$G_r f = f_{ext} \tag{21}$$

where $f_{ext} = \left[\begin{array}{cccccc} 0 & 0 & -mg & 0 & 0 & 0 \end{array} \right]^T$, m is the mass of the platform in kgs and g is the acceleration due to gravity.

V. Motion Planning Algorithm

We now develop the quasi-static motion planning algorithm for the WMR on uneven terrain. As mentioned in the previous sections the planning algorithm is based on the RRT. The adjacent nodes of the RRT are connected using the forward motion problem. Following the methodology proposed in [6] the forward motion problem of a wheeled mobile robot can be formulated into a nonlinear optimization problem subject the kinematic and the force constraints mentioned in the previous sections. The cost function to be minimized is the Peshkin's minimum energy function which is defined as :

$$P_e = -V_{PG}^T[f_{ext} + G_r f^{'}] \tag{22}$$

Where $f^{'}$ are only the components of the tangential forces acting at the contact points. The normal componets of the forces are excluded. Hence the nonlinear optimization problem can now be stated as :
Minimize (22) subject to the kinematic no-slip constraints (10) and (11) for pure rolling and (12) and (11) for pure sliding,the quasi-static equilibrium constraints (21), the friction cone constraints and the constraint on the normal component of the contact forces given by (19) and (20) respectively. The inputs to the optimization routine are the current configuration parameters of the robot with the ground and the wheel ground parameters which satisfy the holonomic constraints given by (14), the vector of joint rates and the coefficient of the coloumb friction, μ, at the contact point of the wheels

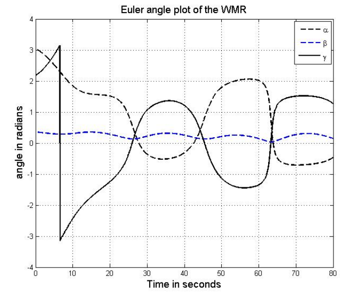

Fig. 6. Euler Angles of the platform center

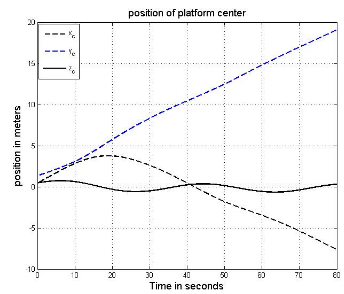

Fig. 7. Position of the platform center

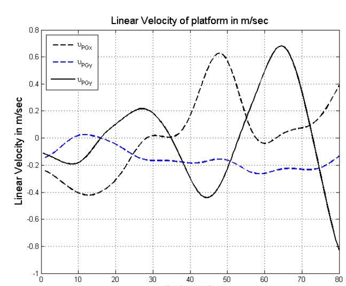

Fig. 8. Linear Velocity of the platform

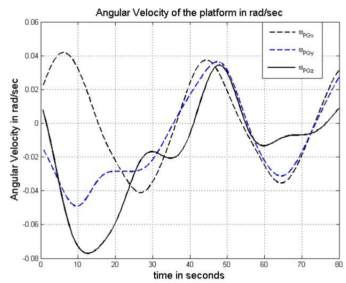

Fig. 9. Angular Velocity of the platform

with terrain. Given the set of joint rates do the following, for each particular set of joint rates do the following:
(i) : Read the current configuration of the robot,$i.e.$, the robot joint variables, the contact variables of the wheels with the ground. For the given joint rates evaluate the contact forces and the velocity of the platform $\{P\}$ $w.r.t$ $\{G\}$ V_{PG} by solving the nonlinear optimization routine with cost function in (22).
(ii): Obtain the contact velocities

$$\begin{bmatrix} v_x^i & v_y^i & v_z^i & \omega_x^i & \omega_y^i & \omega_z^i \end{bmatrix}^T, \forall i = 1,2,3$$

using equation (9). These are the velocities of the frames $\{C_{W_i}\}$ $w.r.t$ $\{C_{G_i}\}$ expressed in body frames $\forall i = 1,2,3$.
(iii) : Using the contact velocties as inputs to Montana's equations of contact intergrate the set of ODEs (13) , (15) and (16) for a time $\delta t = 0.5sec$ to evaluate the next set of contact parameters and the configuration parameters of the robot.
(iv): Repeat steps **(i),(ii),(iii)** for all the set of joint rates and choose the configuration of the WMR closest to the goal point.

VI. Results and Discussion

We develop the plans (trees of the RRT) for the WMR with PVC on a random uneven terrain using the forward motion problem as discussed in the previous section. In these simulations the mass of the platform is taken to be $3kgs$,the acceleration due to gravity is considered to be $g = 9.8m/sec^2$,the coefficient of friction $\mu = 0.5$. The paths computed using the RRT framework essentially involves a search in the joint space of the robot. For the given joint rates the contact velocities and the platform velocities are computed using the forward motion problem discussed in the previous section. This optimization routine is solved using MATLAB's $fmincon$ function. We numerically integrate the ODEs connecting the two adjacent nodes of the RRT using MATLAB's ODE toolbox. We have performed several evolutions of the RRT computed by the planner. One such evolution is shown in the Fig.4. For the sake of illustration on this figure we have superimposed several paths for different starting configurations of the robot.

We show the complete $6dof$ evolution of the vehicle for the path P_1 in Fig.4 in graphs of Figs. 6-9. Fig. 5 shows the snapshots of the WMR on the terrain for path

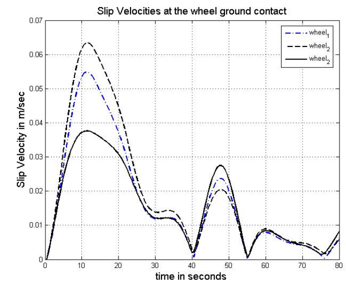

Fig. 10. L_2 norm of the slip velocities

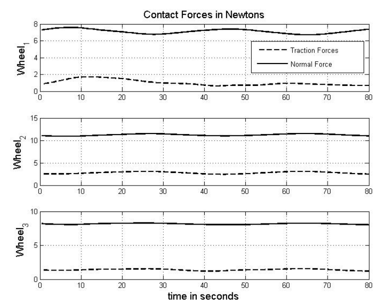

Fig. 11. Wheel Ground traction and normal forces in Newtons

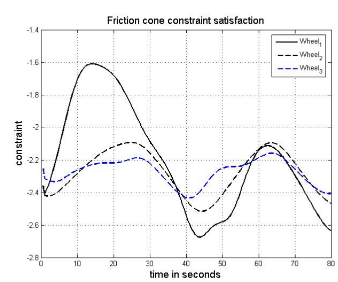

Fig. 12. Friction cone constraint satisfaction

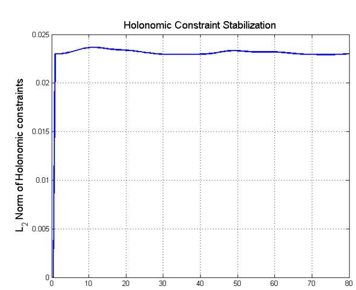

Fig. 13. Holonomic Constraint Stabilization

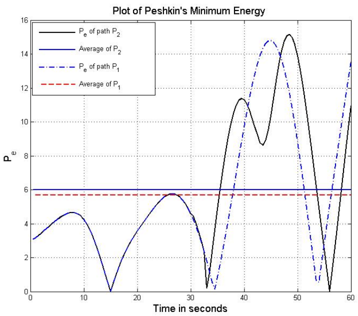

Fig. 14. Peshkin's Minimum Energy in Joules

P_1 in Fig.4. Fig.6 shows the Euler angles corresponding to the $ZYZ(\alpha, \beta, \gamma)$ parameterization of the center of the platform and Fig.7 shows the corresponding position vector $[x_c, y_c, z_c]^T$ of the platform center $w.r.t$ frame G.

Figs 8 and 9 are the values of the linear and angular velocity of the platform obtained as a solution to the optimization routine (the forward motion problem) discussed in the previous section. The L_2 norm of the corresponding wheel ground contact velocities which are calculated using equation (9) are shown in Fig.10. Fig.11 shows the values of the norm of the traction and normal forces at the wheel ground contact points which are obtained as a solution to the forward motion problem. Fig.12 shows the plot of the friction cone constraint which is re-written as

$$\sqrt{f_{xi}^2 + f_{yi}^2} - \mu f_{zi}, \forall i = 1, 2, 3 \qquad (23)$$

The holonomic constraints are stabilized by choosing a suitable value of σ [17] in equation (15). The L_2 norm of the holonomic constraints are shown in Fig.13. Fig.14 shows the plot of the Peshkin's minimum energy for the paths P_1 and P_2 respectively and it can be seen that the path P_1 has a lower average P_e as compared to path P_2. Hence in case of multiple paths to the same goal (intersection of paths P_1 and P_2) it can be concluded that P_1 is more quasi-statically stable than P_2.

VII. Conclusions

The paper presented a quasi-static planner for a wheeled mobile robot for uneven terrains ,having a framework for predicting the $6dof$ evolution of the vehicle platform and its rate of change at every instantwhile satisfying the holonomic and non-holonomic constraints, the permanent contact constraints, kinematic no slip and quasi static constraints at every instant of the evolution. The authors believe that this is possibly the first such effort that provides for satisfaction all the above constraints along with an ability to characterize the $6dof$ evolution of the vehicle platform all within an unified framework. Simulation results based on a RRT framework show the satisfaction of the constraints along with the complete characterization of the robot pose and velocities along a path. Any two nodes of the RRT are connected by a curve evolved by Peshkin's minimum energy principle, which chooses amongst the set of possibilities at a given instant of the evolution, that evolution that has the minimum energy or equivalently one that is most quasi-statically stable. Thus given a set of paths connecting the same goal location and initial configuration the planner can also evaluate, which of the paths has the least integration of Peshkin's minimum energy along the path or that which is most stable.

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