Quasi-Static Simulation of a Wheeled Mobile Robot having a Passive Variable Camber

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Abstract— *In this paper we present an algorithm for quasi-static motion of a wheeled mobile robot equipped with a passive variable camber on uneven terrain. The algorithm is based on Peshkin's minimum energy principle which combines the force and kinematic relationship into a nonlinear optimization problem. The algorithm at each instant estimates the contact forces and velocity of the vehicle platform for a given set of joint velocities of the robot. This ensures that the vehicle satisfies not only kinematic no slip constraints but as well as no slip constraints that arise due to relations between traction and contact forces. In general a complete simulation of a WMR on a fully 3D terrain has been a difficult problem to solve. The best efforts so far have provided a simulation that incorporates the wheel ground contact constraints into a set of differential algebraic equations (DAEs) to estimate the full* $6dof$ *pose of the vehicle. This work integrates the quasi staic contraints within the DAE framework to provide a complete* $6dof$ *evolution of vehicle on 3D terrain that respects both kinematic and quasi static constraints. Simulations that depict variations in evolution of the vehicle with variation in friction coefficients ascertain the validity of the proposed algorithm.*

Keywords: wheeled mobile robots, uneven terrain navigation, quasi-static motion, forward-motion-problem

I. Introduction

To improve mobility of wheeled robots traversing on a fully 3D uneven terrain while maintaining a stable posture is the focus of our research. Most of the research in this area is encouraged from the idea of finding suspension mechanisms for uneven terrain navigation. One of the primary objectives during the design of wheeled robots for uneven terrain is the ability of the robot to navigate with minimum slippage which can be achieved by designing the robot with the essential degrees of freedom that would enhance its ability to negotiate undulations on the terrain. Many such mechanisms have been reported in literature.One such mechanism was proposed by Sreenivasan and Choi [1] wherein two wheels were connected with a variable length axle (VLA) having a prismatic joint to overcome slip during terrain navigation. Chakroborty and Ghosal [2] improved on this mechanism by using a passive variable camber (PVC). More recently Auchter and Carl Moore [3] used ideas from dextrous manipulation of multifingured hands for kinematic modeling of the wheeled mobile robot (WMR) having a PVC. In this work the authors used the instantaneous kinematics of dextrous manipulation developed by Han and Trinkle [5] to derive the kinematic model of the robot. However in these simulations of the WMR with PVC the authors have not incorporated the effect of contact forces and friction cone constraints, which are crucial factors to ensure no-slip during terrain navigation .Also they assume that there exits sufficient friction between the wheel and the terrain to ensure kinemantic no slip. The effect of the friction coefficient which is an essential parameter in ascertaining the traversability of the vehicle on uneven terrain without slip has not been taken in consideration.

In our work we address the issue of quasi-static motion of a WMR equipped with a PVC on uneven terrain and propose an algorithm to perform the forward motion of the WMR. As suggested in [3] the WMR on uneven terrain is analogous to a hand (WMR) grasping an object (ground). We use this analogy in our work to develop the algorithm for the quasi-static forward kinematics of the WMR on uneven terrain and show the effects of the coefficient of friction on the traction force components between the wheel and the terrain. Our analysis is based on the forward object motion problem proposed by Trinkle [4] wherein the kinematic constraints, the quasi-static equilibrium constraints and friction cone constraints are combined into a nonlinear optimization problem using the Peshkin's minimum energy principle. This optimization represents the instantaneous equations of motion of the WMR on uneven terrain. The constraints of this optimization problem are the non-holonomic velocity constraints of the contact points of the wheel $w.r.t$ the ground, the friction cone constraints which ensure kinematic no-slip and the static-equilibrium constraints. The input to this optimzation routine is the vector of joint rates of the robot, the current contact configuration and the effective coefficient of friction between the wheels and the ground. The solution to this optimization routine yields the contact forces and the linear and angular velocties of the robot platform $w.r.t$ the ground. In [6] a force motion planning algorithm was proposed for mul-

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tifingred hand manipulation in that the authors posed the motion planning algorithm as nonlinear optimization problem with pure rolling/sliding constraints, friction cone constraints and static equilibrium constraints. In [7] a quasi-static motion planning algorithm using Peshkin's minimum energy principle was proposed for a multifingured hand manipulating an object. A computational simulation of a fully $6dof$ pose evolution of a vehicle on rough and undulating 3D terrains has proved to be a very challenging problem. Most of the previous methods have only computed an evolution on flat terrains even when the terrain is not flat and is undulating. The idea here has been to compute a kinematically consistent path such as a CC steer assuming the terrain to be flat and hope that a suitable control law would make sure that the vehicle's pitch and roll are somehow adjusted to attain the yaw changes given by the CC steer curve. In other words they are unable to apriori provide a continuous evolution of the fully $6dof$ pose of the vehicle [9],[10] ,[11]. To the best of our knowledge previous work that provides a complete kinematic model of the vehicle that is able to predict continuously the complete evolution of the vehicle can be found in [2],[3]. However these methods have not integrated the quasi static constraints into their differential algebraic equation (DAE) framework. This work integrates the quasi staic contraints within the DAE framework to provide a complete $6dof$ evolution of vehicle on $3D$ terrain that respects both kinematic and quasi static constraints. A testimony towards this is shown in the results section where the current framework is able to predict high slip velocities as the friction coefficient decreases.

II. Problem Statement

Let us consider a 3 wheeled mobile robot having a PVC traversing an uneven terrain as shown in the Fig.1 . The objective of our work is to accomplish the forward motion of the WMR on a random terrain without undergoing slip while maitaining static equilibrium. We assume that the robot has torus wheels. We also assume that the wheels contact the terrain at a single point with Coloumb friction constraints. The Forward motion problem for a wheeled mobile robot is based on the Peshkin's minimum energy principle which simply states that [4] "the system at each instant chooses the easiest motion while satisfying all the constraints". This principle applies to only quasi-static systems subject to forces of constraint (i.e., normal forces arising due to contacts among rigid bodies), Coloumb friction forces and forces independent of velocity.

The Forward motion problem for a three wheeled mobile robot with a passive variable camber can thus be stated as follows

step 1 For the given joint rates of the robot and the contact parameters of the wheels with the ground at the current time step determine the velocity of the platform with respect to the ground and the corresponding contact forces which ensure quasi-static equilibrium. This can be achieved by solving the nonlinear optimization problem which will be explained in later sections.

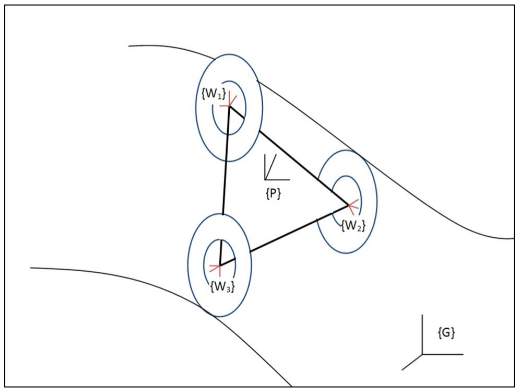

Fig. 1. WMR on uneven terrain

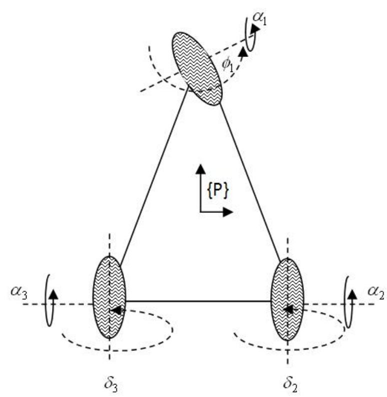

Fig. 2. WMR joints

step 2 After obtaining the platform velocities from step 1 determine the wheel ground contact velocities which will be the input to Montana's kinematic equations of contact [8] and integrate numerically a set of ordinary differential equations (ODEs) which represent the kinematics of the robot (explained in detail in later sections) to obtain the new contact configuration.

Steps **1** and **2** are solved iteratively to obtain the forward motion of the WMR.

III. Kinematics of the WMR

Definitions : For any two reference frames $\{A\}$ and $\{B\}$, $\{R_{AB}, P_{AB}\} \in SE(3)$ is the tranformation matrix of $\{B\}$ $w.r.t$ $\{A\}$, where $R_{AB} \in SO(3)$ is the rotation matrix of the frame $\{B\}$ $w.r.t$ frame $\{A\}$ and $P_{AB} \in R^3$ is the position vector of the origin of frame $\{B\}$ $w.r.t$ $\{A\}$. $SE(3)$ represents the group of special Euclidean transformation matrices and $SO(3)$ the group of rotational matrices.

The velocity vector of the frame $\{B\}$ $w.r.t$ {A}expressed

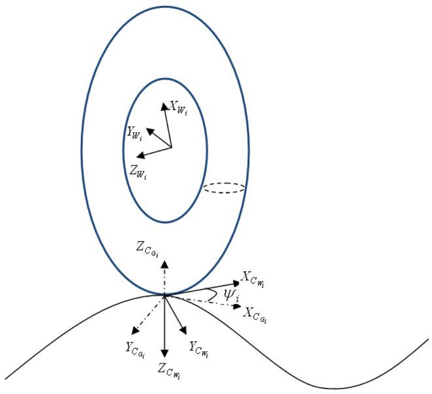

Fig. 3. torus wheel and ground frame assignments

in the body frame ($V^B_{AB} \in R^6$) is given by

$$V^B_{AB} = \begin{bmatrix} \upsilon^B_{AB} \\ \\ \omega^B_{AB} \end{bmatrix} \quad (1)$$

Where $\upsilon^B_{AB} = R_{AB}\dot{P}_{AB}$ and $\omega^B_{AB} = R^T_{AB}\hat{\dot{R}}_{AB}$ $\hat{}$ operator extracts the vector associated with the skew symmetric matrix. For any three reference frames $\{A\}$, $\{B\}$ and $\{C\}$ we have

$$V^C_{AC} = Ad^{-1}_{BC}V^B_{AB} + V^C_{AC} \quad (2)$$

Where the adjoint transformation matrix and its corresponding inverse between frames $\{A\}$ and $\{B\}$ is given by

$$Ad_{AB} = \begin{bmatrix} R_{AB} & R_{AB}\hat{P_{AB}} \\ 0_{3\times3} & R_{AB} \end{bmatrix}$$

$$Ad^{-1}_{AB} = \begin{bmatrix} R^T_{AB} & -R^T_{AB}\hat{P_{AB}} \\ 0_{3\times3} & R^T_{AB} \end{bmatrix}$$

Frame Assignments: Figs. 1 and 3 show the details of the frame assignments we have considered in our analysis. $\{G\}$ is the frame assigned to the ground frame, $\{P\}$ is the frame fixed at the center of mass of the platfrom. $\{C_{G_i}\}$ is the frame fixed on the ground, at the contact point of the i^{th} wheel with the ground. $\{W_i\}$ is the frame assigned to the center of the i^{th} wheel and $\{C_{W_i}\}$ is the frame fixed on the wheel at the contact point. ψ_i is the angle between $x_{C_{G_i}}$ and $x_{C_{W_i}}$. As can be seen from Fig.2 the front wheel (W_1) is steerable, the angle of steer is given by , ϕ_1, and the rear wheels (W_2 and W_3) have a passive variable camber joint whose angles are given by ,δ_2 and δ_3 respectively.
α_i,$\forall i = \{1, 2, 3\}$ is the angle of rotation of the wheels about the $Z_{W i}$ axis.
Velocity relationships:We assume all the velocities expressed in the body frame unless otherwise stated. For the i^{th} kinematic chain we have.

$$V_{PG} = Ad^{-1}_{C_{G_i}G}V_{PC_{G_i}} \quad (3)$$

also we have

$$V_{PC_{W_i}} = Ad^{-1}_{W_iC_{W_i}}V_{PW_i} \quad (4)$$

Also $\{V_{PW_i}\}$ is given by

$$V_{PW_i} = J_{PW_i}(\theta_i)\dot{\theta}_i \quad (5)$$

Where J_{PW_i} is the Jacobian between the platform frame $\{P\}$ and the center wheel $\{W_i\}$ and $\dot{\theta}_i$ is the corresponding joint rate of the i^{th} kinematic chain. Also we have

$$\dot{\theta}_1 = \begin{bmatrix} \dot{\phi}_1 & \dot{\alpha}_1 \end{bmatrix}, \dot{\theta}_2 = \begin{bmatrix} \dot{\delta}_2 & \dot{\alpha}_2 \end{bmatrix}, \dot{\theta}_3 = \begin{bmatrix} \dot{\delta}_3 & \dot{\alpha}_3 \end{bmatrix}$$

For the frames $\{C_{W_i}\}$ and $\{C_{G_i}\}$ we have

$$A_{\psi_i} = Ad_{C_{W_i}C_{G_i}} \quad (6)$$

Where

$$Ad_{C_{W_i}C_{G_i}} = \begin{bmatrix} R_{\psi_i} & 0_{3\times3} \\ 0_{3\times3} & R_{\psi_i} \end{bmatrix}$$

and

$$R_{\psi_i} = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ -\sin\psi_i & -\cos\psi_i & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Using equation (2) for the frames $\{P\}$, $\{C_{W_i}\}$ and $\{C_{G_i}\}$ we have $V_{PC_{G_i}} = A_{\psi_i}V_{PC_{W_i}} + V_{C_{W_i}C_{G_i}}$ and can be re-written as

$$V_{PC_{G_i}} = A_{\psi_i}V_{PC_{W_i}} - V_{C_{G_i}C_{W_i}} \quad (7)$$

$V_{C_{G_i}C_{W_i}}$ represents the relative velocties of the contact frames $\{C_{W_i}\}$ $w.r.t$ $\{C_{G_i}\}$ in the respective body frames. We represent this velocity vector as

$$V_{C_{G_i}C_{W_i}} = \begin{bmatrix} v^i_x & v^i_y & v^i_z & \omega^i_x & \omega^i_y & \omega^i_z \end{bmatrix}^T$$

Using equations (3), (4) ,(5), (6) and (7) we have

$$\begin{bmatrix} v^i_x \\ v^i_y \\ v^i_z \\ \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{bmatrix} = Ad^{-1}_{W_iC_{W_i}}J_{PW_i}(\theta_i)\dot{\theta}_i - A_{\psi_i}V^i_{PG} \quad (8)$$

where $V^i_{PG} = Ad^{-1}_{GC_{G_i}}V_{PG}$. The velocity vector of the frame $\{P\}$ $w.r.t$ $\{G\}$ has

$$V_{PG} = \begin{bmatrix} \upsilon_{PG} \\ \\ \omega_{PG} \end{bmatrix}$$

Where v_{PG} and ω_{PG} are the linear and angular velocity components of the V_{PG} respectively. Further simplifying (8) we have

$$\begin{bmatrix} v_x^i \\ v_y^i \\ v_z^i \end{bmatrix} = R^T_{W_iCW_i} J_{PW_iu}(\theta_i)\dot{\theta}_i - R^T_{W_iCW_i} \hat{P_{W_iCW_i}} J_{PW_il}(\theta_i)\dot{\theta}_i - R^T_{GCW_i} v_{PG} + R^T_{GCW_i} \hat{P_{GCW_i}} \omega_{PG}$$

$$\begin{bmatrix} \omega_x^i \\ \omega_y^i \\ \omega_z^i \end{bmatrix} = R^T_{W_iCW_i} J_{PW_il}(\theta_i)\dot{\theta}_i - R^T_{GCW_i}\omega_{PG} \quad (9)$$

Where $J_{PW_iu}(\theta_i)$ and $J_{PW_il}(\theta_i)$ are the upper and lower partitions of the the Jacobian $J_{PW_i}(\theta_i)$ respectively. For pure rolling we have

$$\begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix} = 0 \quad (10)$$

The constraint that ensures that the wheel does not contact with the terrain is given by

$$v_z^i = 0 \quad (11)$$

The constraints for pure sliding is given by

$$\begin{bmatrix} \omega_x^i \\ \omega_y^i \\ \omega_z^i \end{bmatrix} = 0 \quad (12)$$

A. Montana's kinematics of contact

Let $[u_{w_i}, v_{w_i}, f_w(u_{w_i}, v_{w_i})]^T$ be the parameterization of the contact point on the torus wheel with the ground and $[u_{g_i}, v_{g_i}, f_g(u_{g_i}, v_{g_i})]^T$be the parameterization of the contact point on the ground with the wheel. Also let $\{M_g, K_g, T_g\}$ and $\{M_{w_i}, K_{w_i}, T_{w_i}\}$ be the metric, curvature and the torsion forms of the ground and the i^{th} wheel respectively. Then the variation of the contact parameters $(u_{w_i}, v_{w_i}, u_{g_i}, v_{g_i}, \psi_i)$ $w.r.t$ time is given by Montana's kinematic equations of contact [8]

$$\begin{bmatrix} \dot{u_{w_i}} \\ \dot{v_{w_i}} \end{bmatrix} = M_{w_i}^{-1}K^{-1}(\begin{bmatrix} -\omega_y^i \\ \omega_x^i \end{bmatrix} - K^* \begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix})$$

$$\begin{bmatrix} \dot{u_{g_i}} \\ \dot{v_{g_i}} \end{bmatrix} = M_g^{-1} r_{\psi_i} K^{-1}(\begin{bmatrix} -\omega_y^i \\ \omega_x^i \end{bmatrix} + K_{w_i} \begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix})$$

$$\dot{\psi_i} = \omega_z^i + T_{w_i}M_{w_i}\begin{bmatrix} \dot{u_{w_i}} \\ \dot{v_{w_i}} \end{bmatrix} + T_g M_g \begin{bmatrix} \dot{u_{g_i}} \\ \dot{v_{g_i}} \end{bmatrix}$$

$$0 = v_z^i \quad (13)$$

Where $K = (K_{w_i} + K^*)$ is the relative curvature matrix, $K^* = r_{\psi_i} K_g r_{\psi_i}$ and $r_{\psi_i} = \begin{bmatrix} \cos\psi_i & -\sin\psi_i \\ -\sin\psi_i & -\cos\psi_i \end{bmatrix}$ is the $2D$ representation of the frame $\{C_{W_i}\}$ $w.r.t$ $\{C_{G_i}\}$.

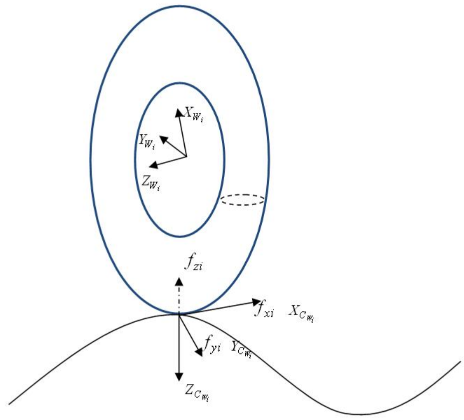

Fig. 4. Forces acting at the contact point of i^{th} wheel

B. Kinematic Equations of the Robot

Using equation (9) as input to (10) we can form fifteen ordinary differential equations (ODEs) $\forall i = \{1, 2, 3\}$. Also the closed loop kinematic chains give rise to a set of constraints know as the holonomic constraints on the robot and ground parameters which can be written as.

$$\{R_{PG}, P_{PG}\}_{W_1} - \{R_{PG}, P_{PG}\}_{W_2} = 0$$
$$\{R_{PG}, P_{PG}\}_{W_1} - \{R_{PG}, P_{PG}\}_{W_3} = 0 \quad (14)$$

These set of algebraic constraints of the form $H(\Theta) = 0$ can be differentiated to a set of ODEs of the form

$$J(\Theta)\dot{\Theta} + \sigma H(\Theta) = 0 \quad (15)$$

where $J(\Theta) = \frac{\partial H}{\partial \Theta}$ and Θ is the set of wheel ground parameters and the robot joint angles. (15) represents the holonomic constraint stabilization equation for DAE systems proposed by Baumgarte [12] and σ is a positive scalar which ensures that the DAE is stable throughout the simulation. The instantaneous degrees of freedom (dof) of the WMR can be found out to be =3 and hence only 3 of the 6 joint variables are actuated which can be chosen as $\dot{\phi_1}$, $\dot{\alpha_2}$ and $\dot{\alpha_3}$.

Hence we have

$$\dot{\theta} = \dot{\theta}_d \quad (16)$$

Where $\theta = \begin{bmatrix} \phi_1 & \alpha_1 & \delta_2 & \alpha_2 & \delta_3 & \alpha_3 \end{bmatrix}^T$ and $\dot{\theta}_d$ are the desired joint rates of the robot.

The equations (13) $\forall i = \{1, 2, 3\}$, (15) and (16) form a set of ODEs which represent the kinematic relationships of the WMR on uneven terrain. These set of equations can be integrated numerically to get the configuration parameters of the system after each time instant.

C. Force-Moment Analysis of the WMR

The forces acting at the $\{C_{W_i}\}$ frame of the i^{th} wheel is shown in the Fig.4. $f_{C_{W_i}} = [f_{xi}, f_{yi}, -f_{zi}]^T$, where

f_{xi}, f_{yi} are the components of the tangential forces acting at the local frame and $-f_{zi}$ is the normal force component acting from the contact point towards the center of the torus cross section. Hence the wrench basis at the $\{C_{W_i}\}$ frame is given by

$$B_{C_{W_i}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The wrench acting at the platform frame $\{P\}$ due to $f_{C_{W_i}}$ is given by

$$F_{PC_{W_i}} = G_{ri} f_{C_{W_i}} \quad (17)$$

Where

$$G_{ri} = \begin{bmatrix} R_{PC_{W_i}} & 0_{3\times3} \\ \hat{P_{PC_{W_i}}} R_{PC_{W_i}} & R_{PC_{W_i}} \end{bmatrix} B_{C_{W_i}}$$

is the grasp matrix of the i^{th} kinematic chain. Hence the total wrench acting on the platform frame $\{P\}$ due to all the contact points is given by

$$F_P = G_r f \quad (18)$$

where the grasp matrix $G_r \in R^{6\times9}$ is given by

$$G_r = \begin{bmatrix} G_{r1}B_{C_{W_1}} & G_{r2}B_{C_{W_2}} & G_{r3}B_{C_{W_3}} \end{bmatrix}$$

and $f = [f_{x1}, f_{y1}, f_{z1}, f_{x2}, f_{y2}, f_{z2}, f_{x3}, f_{y3}, f_{z3}]^T$ is the vector of all the contact forces at each of the contact points. Also as mentioned in the previous section we assume a point contact with coulumb friction at the contact point between the wheel and the terrain. For the vehicle to move without slip at the contact points of the wheels with terrain the forces acting at the contact point must satisfy the friction cone constraints which are given by

$$f_{xi}^2 + f_{yi}^2 \leq \mu^2 f_{zi}^2 \quad (19)$$

Where μ is the coefficient of Coloumb friction at the wheel ground contact points. Also for the wheel to maintain contact with the ground the normal force component at the contact point should be non-negative,$i.e$,

$$f_{zi} \geq 0 \quad (20)$$

For the vehicle to be in quasi-static equilibrium the total wrench on the platform F_P should balance the total external wrench. Hence the quasi-static force-moment balance equation is given by

$$G_r f = f_{ext} \quad (21)$$

where $f_{ext} = \begin{bmatrix} 0 & 0 & -mg & 0 & 0 & 0 \end{bmatrix}^T$, m is the mass of the platform in kgs and g is the acceleration due to gravity.

IV. Forward Motion Algorithm

Now following the methodology proposed in [4] the forward motion problem of a wheeled mobile robot can be formulated into a nonlinear optimization problem subject the kinematic and the force constraints mentioned in the previous sections. The cost function to be minimized is the Peshkin's minimum energy function which is defined as :

$$P_e = -V_{PG}^T [f_{ext} + G_r f^{'}] \quad (22)$$

Where $f^{'}$ are only the components of the tangential forces acting at the contact points. The normal componets of the forces are excluded. Hence the nonlinear optimization problem can now be stated as :
Minimize (22) subject to the kinematic no-slip constraints (10) and (11) for pure rolling (or (12) and (11) for pure sliding),the quasi-static equilibrium constraints (21), the friction cone constraints and the constraint on the normal component of the contact forces given by (19) and (20) respectively. The inputs to the optimization routine are the current configuration parameters of the robot with the ground and the wheel ground parameters which satisfy the holonomic constraints given by (14), the vector of joint rates and the coefficient of the coloumb friction, μ, at the contact point of the wheels with terrain. The forward motion algorithm can be summarized as follows.

(i) : Read the current configuration of the robot,$i.e.$, the robot joint variables, the contact variables of the wheels with the ground. For the given joint rates evaluate the contact forces and the velocity of the platform $\{P\}$ $w.r.t$ $\{G\}$ V_{PG} by solving the nonlinear optimization routine (21).

(ii): Obtain the contact velocities

$$\begin{bmatrix} v_x^i & v_y^i & v_z^i & \omega_x^i & \omega_y^i & \omega_z^i \end{bmatrix}^T, \forall i = 1, 2, 3$$

using equation (9). These are the velocities of the frames $\{C_{W_i}\}$ $w.r.t$ $\{C_{G_i}\}$ expressed in body frames $\forall i = 1, 2, 3$.

(iii) : Using the contact velocties as inputs to Montana's equations of contact intergrate the set of ODEs (13) and (15) for a time period $\delta t = 0.05sec$ to evaluate the next set of contact parameters and the configuration parameters of the robot.

Repeat steps **(i),(ii),(iii)** iteratively to obtain the forward motion of the robot on uneven terrain.

V. Simulations and results

We show simulation results of the WMR with PVC on a random uneven terrain using the forward motion algorithm for several values of coefficients of friction μ and actuator rates $\dot{\theta}_d$.

Fig.5 shows the snapshots of the quasi-static motion of the WMR on a random terrain. In this simulation the mass

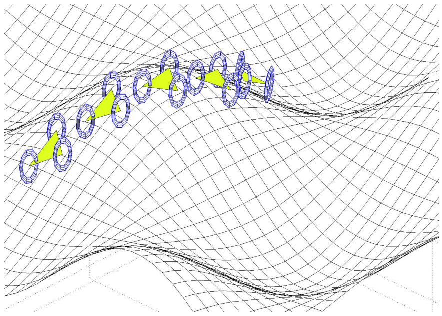

Fig. 5. WMR simulation on random uneven terrain

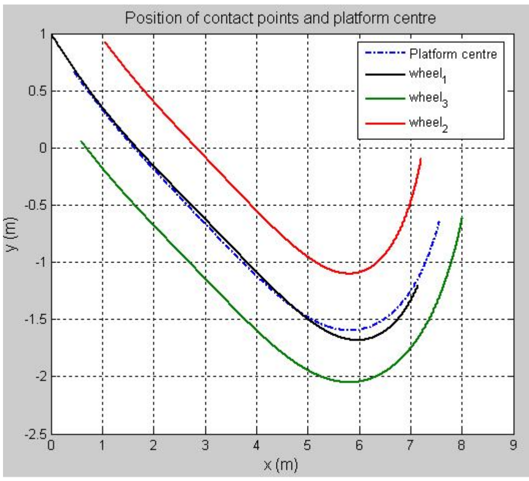

Fig. 6. Locus of wheel ground contact points and the center of the platform

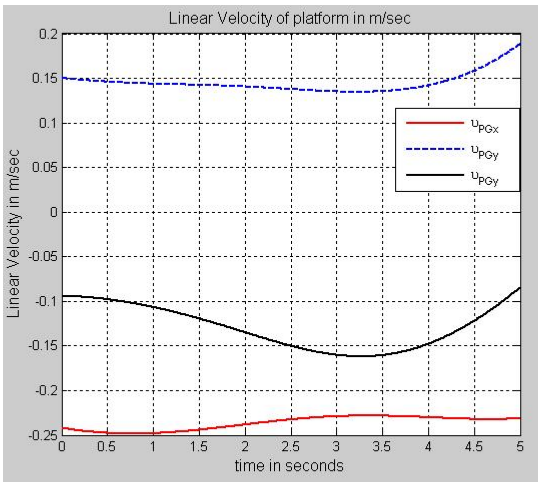

Fig. 7. Components of the Linear Velocity of the platform

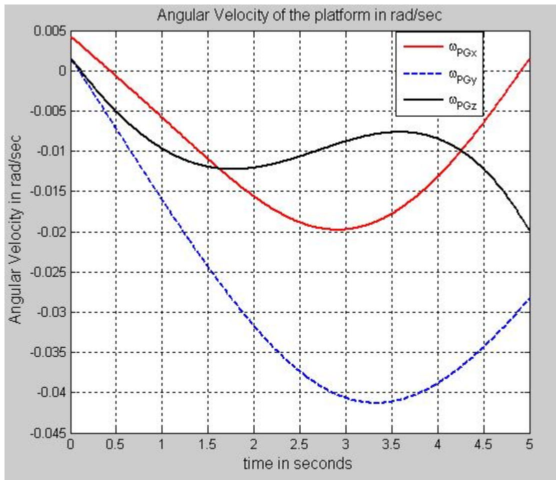

Fig. 8. Components of the Angular Velocity of the platform

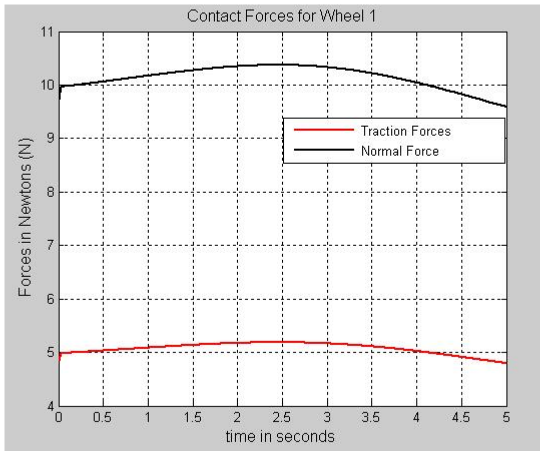

Fig. 9. Normal and Traction force components on Wheel1

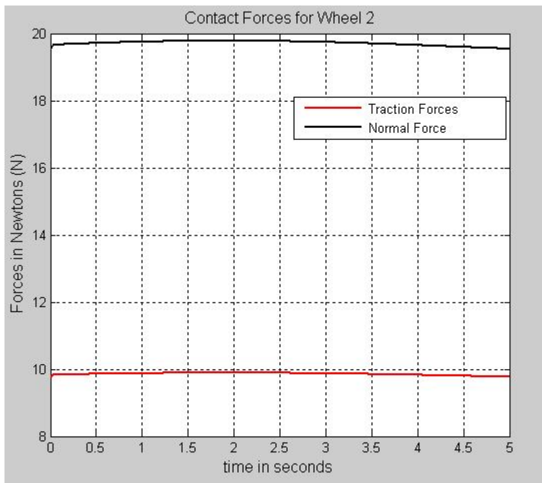

Fig. 10. Normal and Traction force components on Wheel2

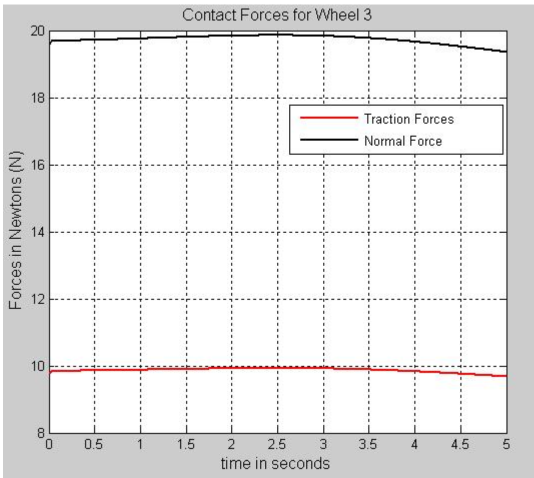

Fig. 11. Normal and Traction force components on Wheel3

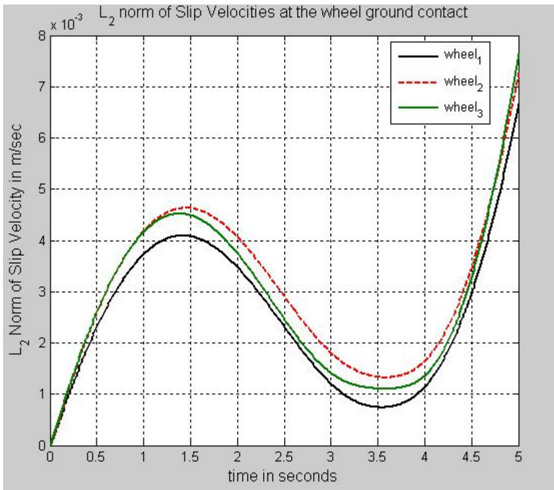

Fig. 12. L_2 norm of Slip Velocities at the wheel ground contact points

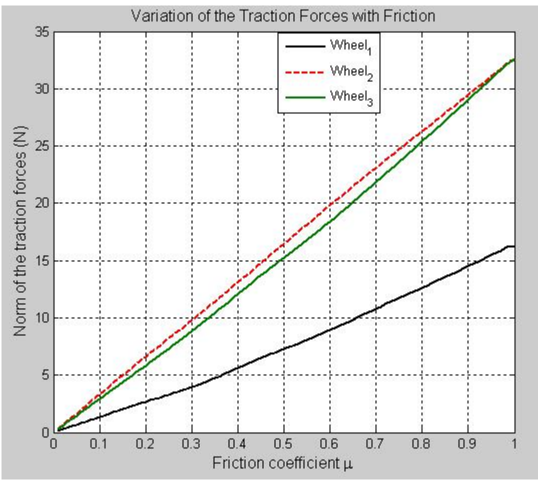

Fig. 13. Variation of Traction Forces with friction coefficient μ

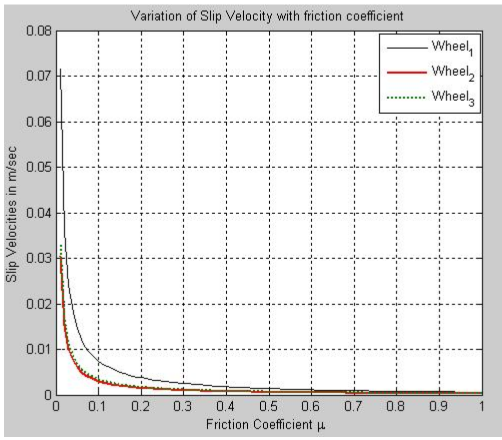

Fig. 14. Variation of Slip Velocities with friction coefficient μ

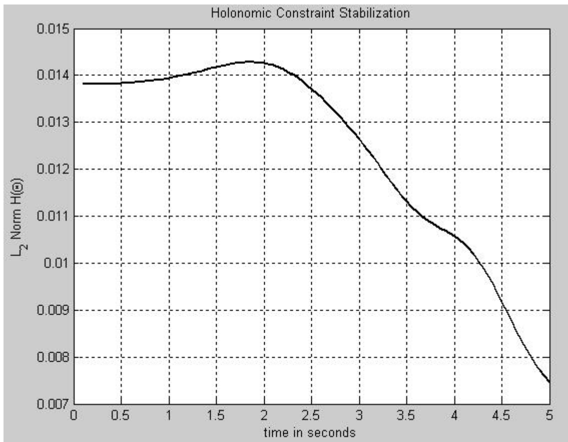

Fig. 15. Holonomic Constraint Stabilization

of the platform is taken to be $5kgs$, the acceleration due to gravity is considered to be $g = 9.8m/sec^2$, the coefficient of friction $\mu = 0.5$ and$\dot{\theta_d} = [0, 0.6, 0, 0.6, 0, 0.6]^T$. Fig.6. shows the locus of the wheel ground contact points and the center of the platform for the random terrain in Fig.4. Figs 7 and 8 show the linear and angular velocities of the platform in m/sec and rad/sec respectively. Fig.9,10 and 11 shows the components of the contact forces at each wheel ground contact points . Fig.12. shows the L_2 norm of the slip velocities at the wheel ground contact points. As mentioned in the previous sections the algorithm is able to estimate the effects of coefficients of friction on the traction force components at the wheel ground contact points. Fig.13 shows the variation of traction force components with μ. Fig.14 shows the variation of slip velocities with μ. As is evident from the plots the algorithm estimates the traction and slip velocities at the point of contact. Fig.15 shows the stabilization of the holonomic constraints $H(\Theta)$ for a positive value of σ.

VI. Conclusions and Future Work

In this paper we have presented the Forward Motion algorithm for a wheeled mobile robot with PVC. The algorithm implements the quasi-static forward motion of the WMR on uneven terrain.The main contribution here is the coupling of the quasi static constraints into the DAE framework that computes the evolution of the WMR on uneven terrain. The consequence of this is a fully $6dof$ pose evolution that respects the quasi-static constraints. To the best of our knowledge such a methodology that integrates both kinematic and quasistatic constraints into a single unified framework for a WMR does not seem to have appeared before in literature. The literature that concerns with evaluation of satisfaction of quasi static stability, friction cone constraints either assume the pose of the robot is known or use some form of approximation or heuristic to estimate its pose along a trajectory. While those that rigorously compute a continuous $6dof$ evolution of the vehicle have not verified whether the evolution is indeed possible from other aspects of stability mentioned above. The efficacy of the proposed integrated framework is seen in its ability to show the increase in slip velocities and reduction in traction forces with a reduction in friction coefficient. Such an ability would not be possible for WMR that evolve purely on kinematic principles alone . We plan to extend our results to quasi-static inverse kinematics of the WMR and also develop a motion planning algorithm which can cause the robot to move from any initial configuration to a desired final configuration while maintaining static-equilibrium and no-slip.

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