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Optimum Steering input determination and Path-Tracking of All-Wheel Steer Vehicles on Uneven Terrains based on Constrained Optimization.

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Abstract— **In this paper we propose a framework for optimum steering input determination of all-wheel steer vehicles (AWSV) on rough terrains. The framework computes the steering input which minimizes the tracking error for a given trajectory. Unlike previous methodologies of computing steering inputs of car-like vehicles, the proposed methodology depends explicitly on the vehicle dynamics and can be extended to vehicle having arbitrary number of steering inputs. A fully generic framework has been used to derive the vehicle dynamics and a non-linear programming based constrained optimization approach has been used to compute the steering input considering the instantaneous vehicle dynamics, no-slip and contact constraints of the vehicle. All Wheel steer Vehicles have a special parallel steering ability where the instantaneous centre of rotation (ICR) is at infinity . The proposed framework automatically enables the vehicle to choose between parallel steer and normal operation depending on the error with respect to the desired trajectory. The efficacy of the proposed framework is proved by extensive uneven terrain simulations, for trajectories with continuous or discontinuous velocity profile.**

I. INTRODUCTION

All- wheel steer vehicleic vehicles are increasingly being used in outdoor applications because of their increased maneuverability. All-wheel steer-vehicles such as ATHLETE [1] and MIT rover [2] possess excellent maneuverability over rough terrain. Although a lot of work has been done on controlling and determining steering inputs for a car-like vehicle, the extension of these frameworks for all-wheel steer vehicles is not trivial and even in some cases inappropriate.

The focus of the paper is divided into two parts: Developing a framework to determine steering inputs for a all-wheel steer vehicles considering the vehicle dynamics in 3D and the desired path parameters and computing a feasible motion profile for the vehicle to return to the desired trajectory, once deviated, without violating the no-slip and contact constraints. In particular here we extend our earlier framework [12] for trajectory generation to the path tracking domain. One of the most popular methodologies for determining steering input for a car like vehicle can be found in [3]. The pure-pursuit methodology as it is called is a geometric approach which consists of calculating the curvature of a circular arc that connects the midpoint of the rear axle to a goal point on the path ahead of the vehicle and uses a pre-specified look-ahead distance to calculate the steering angle. But this methodology does not explicitly include the vehicle dynamics while computing the steering angles .It comes intuitively that for a vehicle operating on rough terrain steering inputs will have significant effect on the vehicle dynamics and path tracking ability as well. Author in [4] proposed the framework for determining steering input for a double-steered vehicle considering lateral forces acting on the wheel based on vehicle-side slip angle. But the extension of this methodology for all-wheel independently steered vehicle is not trivial. In [2] and [5] authors use a methodology of determining steering input by mapping desired vehicle velocities to steering joints velocities. Both [2] and [5] does not talk about existence of unique instantaneous centre of rotation (ICR) which is essential for an independently steered vehicle. Moreover since the methodology is at kinematics level, it does not capture the effect of vehicle dynamics on steering inputs. Previous path tracking methodologies for differentially steered vehicles can be found in [6, 7, 8, 9] .The core all the path tracking methodologies consists of designing controllers which will drive the vehicle along a particular trajectory. But in all the above approaches the governing equations of the vehicle upon which the controller design is based upon is at the kinematics level i.e dependent upon the wheel rotation parameters such as wheel angular velocity. A mapping is generally done to relate the wheel angular velocity to the vehicle chassis velocity. For a vehicle evolving on rough terrain, wheel slip will be an irremovable parameter and the assumption of constraining the wheel to roll without slip seems to be untenable. Hence the goal of the proposed work is to frame the dynamics of the vehicle in such a manner so as to control the velocity of the chassis considering the forces and torques as the input to the system rather than wheel rotation parameters. Previous attempt to control velocities considering forces and torques as the input can be found in [10] where Cartesian velocities were controlled by computing wheel motor torques.

The proposed approach builds upon that idea but contributes by taking into account the terrain conditions and vehicle dynamics in 3D. Moreover the framework can be applied to any generic wheeled vehicle having arbitrary number of steering inputs. We don't attempt to model the wheel slip and the friction characteristics of the terrain as these are highly terrain dependent and model derived for one terrain condition may prove to be of little use on others. We show with the help of simulations that by utilizing the inherent redundancy that an all-wheel steer vehicle provides, it is possible to maintain efficient path tracking even on uneven terrain with the proposed framework. Carefully designed all-wheel steer vehicles have omni-directional ability and hence

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fall in the holonomic vehicles category. Such vehicles have 3 dof in the yaw plane and the vehicle's longitudinal and lateral velocities are independent of the heading direction which makes application of only position level tracking possible.

The proposed framework converts the path following problem of the vehicles into a non-linear optimization problem which computes the maximum possible velocity and acceleration that the vehicle can generate, depending on the terrain condition and vehicle posture to return back once deviated from the trajectory. Corresponding to that maximum velocity and acceleration the optimum wheel motor torques and steering input is determined. The proposed work maps the position level error into force and torque space. Efficacy of the proposed framework is shown by extensive simulations on uneven terrains.

The rest of the paper is organized as follows: Section II outlines the problem formulation and basic idea behind the current work. Section III describes the derivation of vehicle dynamics. Section IV describes the optimization routine for determining the steering inputs. Section V presents the simulation results for the proposed framework.

II. Problem Formulation and Basic Idea

Path tracking of passive suspension vehicle is essentially a planar problem in the sense that it deals with controlling the motion of the vehicle in the yaw plane. However when a vehicle is moving over a general uneven terrain in 3D , the dynamics no-longer remain constrained in a plane and hence to control the yaw plane motion of the vehicle, the general dynamics in 3D has to be taken into account. The vehicle's state is described in terms of position x, y, z and orientation variable i.e pitch angle (γ), roll angle (β) and the heading angle (α). We define a subset of the above comprising of the parameters that are to be controlled as

$$q_s = \begin{bmatrix} x & y \end{bmatrix}^T \tag{1}$$

The yaw angle is left out because omni-directional ability of the vehicle makes only position level tracking possible. Variables other than these are passive and their evolution will depend on x, y, α and their derivatives.

Figure 1(a) shows the top view of the vehicle (yaw plane view) and the reference path. We take here the XY plane as the yaw plane. We attach two reference frames $\{G\}$ and $\{L\}$ to the centre of the mass of the vehicle. The former is a fixed frame represented by subscript G which moves with the chassis but is fixed in the orientation, while the later is the body reference frame represented by subscript L.The desired trajectory is parametrized with respect to time.

We define the vehicle's current(q_{ct}) and desired state (q_{dt}) at time t as:

$$q_{ct} = \begin{bmatrix} x_c(t) & y_c(t) \end{bmatrix}^T \tag{2}$$

$$q_{dt} = \begin{bmatrix} x_d(t) & y_d(t) \end{bmatrix}^T \tag{3}$$

$$e_x(t) = x_d(t) - x_c(t), \quad e_y(t) = y_d(t) - y_c(t) \tag{4}$$

The Cartesian error $e_x(t)$, $e_y(t)$ decides the motor torque and steering input. The objective is to minimize them at every instant.

We now describe the basic idea behind the proposed path tracking methodology. Figure 1(b) shows a generic 4-wheeled independently steered vehicle. The generalized forces acting on the vehicle are the wheel ground normal contact forces N_i and traction forces resulting from the wheel motor torques T_i acting along the unit vector $\hat{t_i}$ .The vehicle's motion is controlled by these traction force vectors. Figure 1(c) shows an equivalent scenario where a rigid body in a plane is acted upon by four forces at different angles. The velocity of the rigid body can be controlled by resolving them along the X and Y direction and coupling them with equation (4) as

$$F_x = |F_1|\cos\theta_1 + |F_2|\cos\theta_2 - |F_3|\cos\theta_3 - |F_4|\cos\theta_4 \tag{5}$$

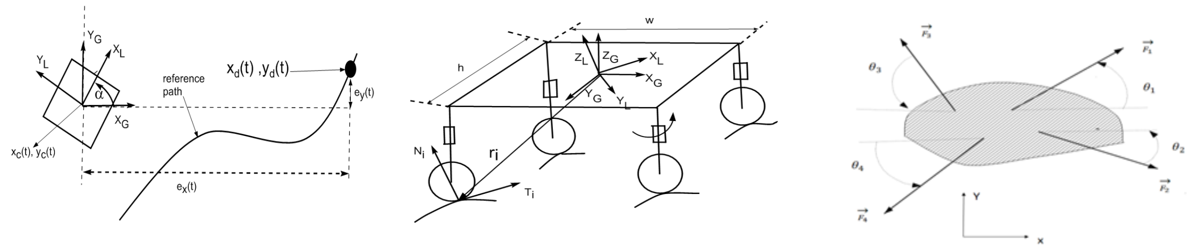
$$F_y = |F_1|\sin\theta_1 + |F_2|\sin\theta_2 - |F_3|\sin\theta_3 - |F_4|\sin\theta_4 \tag{6}$$

$$F_x = K_p e_x(t) + K_v \dot{e_x}(t), \quad F_y = K_p e_y(t) + K_v \dot{e_y}(t) \tag{7}$$

Situations shown in figure 1(b) and figure 1(c) can be seen to be analogous if we approximate the forces to be equivalent to the traction forces acting on the vehicle and the angles at which the forces are acting to be the steering input of the vehicle. In equation (5) and (6) if the angles are kept constant and the motion is controlled by varying the force input, then the resulting system will be similar to that of a skid-steered vehicle where the vehicle's motion is controlled by the variation of the wheel motor torques. But if the forces as well as the angles are allowed to vary, then the resulting system will be that of a 4-wheeled independently steered vehicle.

However, there is one critical difference between the two situations. Equation (5) and (6) suggests that, when deviated from the desired trajectory, the object will try to converge back to it. The magnitude of velocity and acceleration with which the convergence happens depends upon the amount of deviation and constants K_p and K_v which can be thought to be similar in nature to proportional and derivative constants. For a free moving body in space like an aircraft, the maximum velocity and acceleration depends upon only on the actuator limits and hence considering ideal actuators any arbitrary velocity and acceleration could be achieved. However, for a vehicle operating on uneven terrain, the maximum acceleration limit depends upon the terrain condition, the current posture and linear and angular acceleration. High velocities on uneven terrains will result in vehicle losing contact with the ground or violating no-slip (friction cone) constraints because vehicle's wheel ground contact forces are a function of vehicle velocity and acceleration. So the methodology described in (5) and (6) cannot be directly applied in case of a vehicle. We resolve this issue by searching for the maximum possible feasible acceleration, producing forces as close as possible to the required value given by (7) and corresponding to those forces optimized values of wheel motor torques and steering input are searched which seeks to minimize the Cartesian trajectory error define in (4).

In the subsequent sections we derive the vehicle dynamics in 3D and conditions for the unique Instantaneous Centre of



(a) (b) (c)

Fig. 1. (a) Path tracking and Cartesian error defintion. (b) A generic 4 wheeled independently steered vehicle. (c) A rigid body acted by four forces in a plane.

Rotation (ICR) at each instant of the motion.

III. Vehicle Dynamics

The framework followed here has been derived with the help from authors previous works [12]. It consists of the following main parts: (1) Calculating the unit vectors of the external forces acting on the system. (2) Deducing the conditions for the existence of unique ICR. (3) Solving for the wheel motor torques and steering inputs as a constrained optimization approach.

At each wheel ground contact point we define a contact frame described by the normal-force unit vector $\hat{n_i}$ traction force unit vector $\hat{t_i}$ and the lateral force unit vector $\hat{l_i}$ where the subscript $'i'$ denotes the i^{th} wheel (figure 1(b)). The normal contact force unit force vector can be obtained online if the vehicle is equipped with wheels as proposed in [11] or analytical expressions for them can be obtained if the surface equation is known as in [12]. In simulation we obtain it through a high fidelity physics engine. With the known information of $\hat{n_i}$ the traction force unit vector can be obtained in the following manner.

A. Calculation of Traction Force unit Vector

Let μ_i^* represent the i^{th} wheel axis unit vector which for a non-steered vehicle will be orthogonal to vehicle sagittal plane and and for the frame assignment shown in figure 1(b) can be written as

$$\hat{\mu_i^*} = R \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \tag{8}$$

where R is the rotation matrix describing the orientation of frame $\{L\}$ with respect to $\{G\}$ (ref. figure 1(b)). To get the wheel axis unit vector under the effect of steering the vector equation (8) needs to be rotated about the steering axis by the steering angle. The steering axis in global frame is given by

$$\hat{\delta_i} = R \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \tag{9}$$

The wheel axis unit vector, under the effect of steering can be written with the help of steering axis vector $\hat{\delta_i}$ as

$$\mu_i = \hat{\mu_i^*} \cos\phi_i + \sin\phi_i (\hat{\delta_i} \times \hat{\mu_i^*}) + (1 - \cos\phi_i)(\hat{\delta_i}.\hat{\mu_i^*})\hat{\delta_i} \tag{10}$$

where ϕ_i is the steering angle input of the i^{th} wheel considering anticlockwise rotation as positive.

The traction force unit vector can de deduced from the wheel axis unit vector as.

$$\hat{t_i} = \frac{\hat{\mu_i} \times \hat{n_i}}{|\hat{\mu_i} \times \hat{n_i}|} \tag{11}$$

It can be seen from (10) and (11) that the traction force unit vectors are a function of steering angles.

B. Conditions for Unique ICR

Instantaneous Centre of Rotation for a independently steered vehicle refers to the point of intersection of wheel rotation axis. While intersection of two wheel rotation axis defines an ICR, intersection of all wheel rotation axis defines an unique ICR which is necessary for the vehicle to navigate without skidding. ICR for a vehicle navigating over uneven terrain is not constrained to lie in a plane.

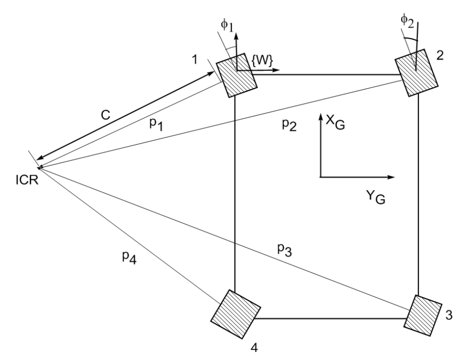


Fig. 2. Definition of ICR

For deducing the conditions for the vehicle to have a unique ICR we attach a reference frame $\{W\}$ at the centre of the first wheel. $\hat{p_1}^W, \hat{p_2}^W, \hat{p_3}^W, \hat{p_4}^W$ represents the wheel rotation axis in the reference frame $\{W\}$ which has the same orientation as $\{G\}$ and attached to the centre of the first wheel. C locates the ICR with respect to $\{W\}$. As can be seen from figure 2 we have

$$\hat{p_1}^W = \mu_1 \tag{12}$$

Coordinate of the ICR can be represented with the help of (12) as

$$\chi_{ICR} = \left[\begin{array}{ccc} C\hat{p_{1x}}^W & C\hat{p_{1y}}^W & C\hat{p_{1z}}^W \end{array} \right]^T \quad (13)$$

Coordinates of the wheels from the frame $\{W\}$ can be written as

$$\chi_1 = R \left[\begin{array}{ccc} 0 & 0 & 0 \end{array} \right]^T \quad (14)$$

$$\chi_2 = R \left[\begin{array}{ccc} -w & 0 & -(d_1 - d_2) \end{array} \right]^T \quad (15)$$

$$\chi_3 = R \left[\begin{array}{ccc} -w & -h & -(d_1 - d_3) \end{array} \right]^T \quad (16)$$

$$\chi_4 = R \left[\begin{array}{ccc} 0 & -h & -(d_1 - d_4) \end{array} \right]^T \quad (17)$$

where d_i refers to the i^{th} leg length connecting the wheel to the chassis. h, w are defined as shown in figure 1(b) Vectors $\hat{p_2}^W, \hat{p_3}^W, \hat{p_4}^W$ can be written in terms of ICR coordinate as

$$\hat{p_2}^W = \frac{\chi_{ICR} - \chi_2}{|\chi_{ICR} - \chi_2|} \quad (18)$$

$$\hat{p_3}^W = \frac{\chi_{ICR} - \chi_3}{|\chi_{ICR} - \chi_3|} \quad (19)$$

$$\hat{p_4}^W = \frac{\chi_{ICR} - \chi_4}{|\chi_{ICR} - \chi_4|} \quad (20)$$

Steering angles ϕ_i can be found as the angle between wheel axis unit vector $\hat{p_i}^W$ and $R \left[\begin{array}{ccc} 1 & 0 & 0 \end{array} \right]^T$

$$\phi_2 = acos(\hat{p_2}^W.R \left[\begin{array}{ccc} 1 & 0 & 0 \end{array} \right]^T) - \frac{\pi}{2} \quad (21)$$

$$\phi_3 = acos(\hat{p_3}^W.R \left[\begin{array}{ccc} 1 & 0 & 0 \end{array} \right]^T) - \frac{\pi}{2} \quad (22)$$

$$\phi_4 = acos(\hat{p_4}^W.R \left[\begin{array}{ccc} 1 & 0 & 0 \end{array} \right]^T) - \frac{\pi}{2} \quad (23)$$

Equations (12)-(23) relates ϕ_1 and C to ϕ_2, ϕ_3, ϕ_4 for a given unique ICR. C locates the coordinates of the ICR and varies according to the error with respect to the desired trajectory. The proposed method of relating ICR to the steering input has a distant similarity to [15]. However the approach described in this paper is more suited for uneven terrain navigation where the wheel rotation axis can lie in 3D space. For an all-wheel steer vehicle $C = infinity$ corresponds to the special condition called the parallel steer mode. The framework described later in this section searches for an optimum value of the C given a particular trajectory error hence enabling an automatic switching between normal and parallel steer operation.

C. Equations of Motion of the Vehicle

The equations of motion of an all wheel steer vehicle can be written as

$$\sum_{i=1}^{4} N_i.\hat{n_i} + \sum_{i=1}^{4} T_i.\hat{t_i} = \left[\begin{array}{ccc} F_x & F_y & F_z \end{array} \right]^T \quad (24)$$

$$\sum_{i=1}^{4} r \times N_i.\hat{n_i} + \sum_{i=1}^{4} r \times T_i.\hat{t_i} = \left[\begin{array}{ccc} M_x & M_y & M_z \end{array} \right]^T \quad (25)$$

$$\left\{ \begin{array}{lll} F_x = ma_{dx}, & F_y = ma_{dy}, & F_z = ma_{dz} \\ M_x = I_{xx}\dot{\omega}_x, & M_y = I_{yy}\dot{\omega}_y, & M_z = I_{zz}\dot{\omega}_z \end{array} \right. \quad (26)$$

a_{dx} and a_{dy} are maximum possible accelerations to be determined from the optimization framework described in the next section. Instantaneous value of a_z which is the acceleration along the gravity direction can be provided by the inertial sensors. Through the inertial sensors we can also obtain $\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z$ which are the angular accelerations along the roll, pitch and yaw axis respectively. In the course of simulation the information required from the inertial sensors are provided by a physics engine.

I_{xx}, I_{yy}, I_{zz}, m are respectively the vehicle chassis moment of inertia and mass. $\vec{r_i}$ is the radius vector from the centre of the mass of the vehicle to the i^{th} wheel ground contact point(figure 1(b)). (24) and (25) can be written in a matrix form as

$$A * B = D \quad (27)$$

where $B = \left[\begin{array}{cc} T_i & N_i \end{array} \right]^T_{8\times1}$ $D = \left[\begin{array}{cccccc} F_x & F_y & F_z & M_x & M_y & M_z \end{array} \right]^T_{6\times1}$. A is a matrix dependent on the vehicle geometry, posture and normal contact force distribution, and steering angles.

IV. OPTIMIZATION ROUTINE

Equation (27) along with (21)-(23) represents 11 non-linear equations in terms of 15 variables. They are 4 T_i and N_i each, 4 steering angles ϕ_i, coordinates of ICR (since the steering angles are themselves function of the ICR coordinates) , a_{dx} and a_{dy}. N_i are also taken as variables as these depends upon vehicle velocity and acceleration as stated earlier.

We solve equation (27) and (21)-(23) as a constrained optimization problem with the following inequality constraints.

$$N_i > 0 \quad (28)$$

$$|(T_i)| < \rho N_i \quad (29)$$

$$\tau_i^{min} \geq (T_i).r \geq \tau_i^{max} \quad (30)$$

$$\phi_{min} \geq \phi_i \geq \phi_{max} \quad (31)$$

(28) corresponds to the constraint that the wheels always maintains contact with the ground. (29) corresponds to the friction cone constraint. (30) corresponds to the constraint that wheel torque required to generate the traction forces is between τ_i^{min} and τ_i^{max} and (31) represents the steering angle limit.

The objective function is framed as

$$J = u^T u \quad (32)$$

$$u = \left[\begin{array}{c} a_{dx} - (K_p e_x(t) + K_v \dot{e_x}(t)) \\ a_{dy} - (K_p e_y(t) + K_v \dot{e_y}(t)) \end{array} \right]$$

The objective function seeks to bring the commanded acceleration as close as possible to the required acceleration. The non-linear optimization is solved using MATLABs in-built function FMINCON.

V. SIMULATION RESULTS

Extensive simulations were performed using high fidelity physics engine MSC Visual NASTRAN and MATLAB with $m = 10kg, \rho = 0.7, K_p = 80, K_v = 18, K_{st} = 180$.The

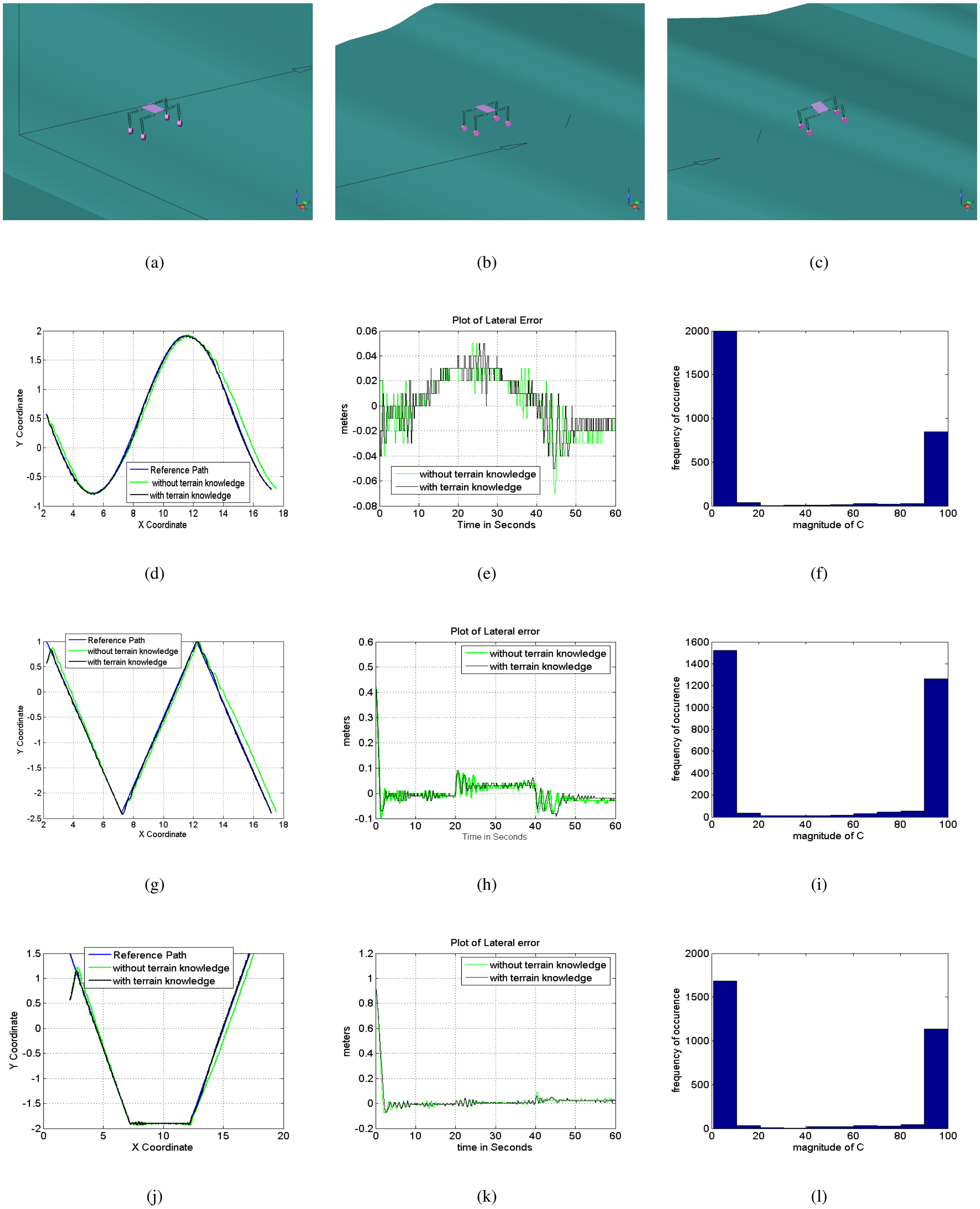


Fig. 3. (a)-(c) Vehicle's Evolution on Uneven Terrain. (d) Tracking Performance for a smooth sinusoidal trajectory. (e) Lateral error associated with tracking smooth sinusoidal trajectory. (f) Frequency with which a particular value for coordinates of iCR, C was chosen while tracking while tracking sinusoidal trajectory. (g) Tracking Performance for a saw tooth like curve. (h) Lateral error associate with tracking saw tooth curve. (i) Frequency with which a particular value for coordinates of ICR, C was chosen while tracking saw tooth curve. (j) Tracking Performance for another trajectory with discontinuous velocity profile. (k) Lateral error associated with tracking curve shown in (j). (l) Frequency with which a particular value for coordinates of ICR, C was chosen while tracking the trajectory.

terrain used in the simulation were moderately uneven. Figure 3(a) - figure 3(c) shows the evolution of the vehicle on the uneven terrain (please refer to the video[14]).

A. Continuous Trajectories

Figure 3(d) shows the tracking performance for the continuous sinusoidal path. The initial position of the vehicle was synchronized with the starting point of the trajectory. The tracking performance is shown for two different cases depending on whether the vehicle has the terrain knowledge or not. The terrain knowledge here refers to the surface contact normal information which is used to derive the vehicle dynamics. The lateral tracking error variation with time is shown in figure 3(e). It can be seen that with the current framework the motion of the omni-directional vehicle can be controlled to maintain fairly small tracking error even without terrain knowledge. Figure 3(f) shows a histogram plot of the location of the ICR in terms of C. The plot shows the frequency with which C values in a particular region is chosen. A high value of C close to 100 signifies the parallel steer mode which occurs when the lateral error is significant. For smaller lateral errors smaller value of C is preferred. From the definition of C in section IV, a smaller value of C would mean a smaller steering angle which would indeed occur only when the lateral error is small. The path considered in this case however is continuous which does not challenge much the capabilities of the omni-directional vehicle. Hence we present, next the tracking performance for two trajectories having discontinuous velocity profile.

B. Trajectory with discontinuous Velocity Profile 1

Figure 3(g) shows the tracking performance for a saw-tooth like curve. In this case the initial position of the vehicle was offset laterally from the starting point of the trajectory by around 0.6m. The initial reaction of the vehicle is to move to parallel steer mode and converge as fast as possible to the desired trajectory. The initial lateral error in figure 3(h) is high due to the fact that vehicle starts from an offset from the desired trajectory but settles down to a low value eventually. An important difference between the tracking of this and the previous trajectory is the variation of C. Figure 3(i) shows that the frequency of occurrence of the parallel steer mode ($C \approx 100$) has considerably increased for this trajectory. This is mainly induced by the large lateral error at the start of the simulation because of the offset and rapid variation of velocity profile because of discontinuity.

C. Discontinuous Trajectory 2

Figure 3(j) shows the tracking performance for another trajectory with discontinuous velocity profile . The tracking difficulty of this curve lies between the smooth sine curve and saw tooth curve. The desired trajectory in this case consists of a straight line portion parallel to X axis which is easier to track and hence the lateral error, shown in figure 3(k) in this case is less than that encountered for the saw tooth curve. However the fact that the curve still has a discontinuous velocity profile is the reason for the lateral error to be more than the smooth sine curve. The histogram plot of C in figure 3(l) confirms similar trend where the frequency of occurrence of parallel steer mode lies between the sine curve and saw tooth curve.

VI. CONCLUSIONS

In this paper we presented a novel method based on constrained optimization for determination of steering input for All Wheel Independently Steered Vehicles. Given the fact that these type of vehicles are viewed as solutions for planetary exploration, the presented framework provides the necessary framework for controlling these vehicles on uneven terrain. The presented work went beyond existing vehicle control framework by explicitly bringing the vehicle's stability constraints such as permanent contact and no-slip within the control framework. The optimum steering input determination based on tracking error allows the vehicle to switch automatically between normal and parallel steering mode and this feature was shown to be very critical for maintaining small tracking error.

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