Localizing from Multi-Hypotheses States Minimizing Expected Path Lengths for Mobile Robots

Hemanth K, Subhash S and K. Madhava KRISHNA*

Robotics Research Center, IIIT Hyderabad,
India
**E-mail: mkrishna@iiit.ac.in*
www.iiit.net

Amit K Pandey

Group: RIA, LAAS-CNRS
Toulouse, France
E-mail: akpandey@laas.fr

This paper explains how to singularize a robot to a unique hypothesis state from a state of multiple hypotheses. This it does by computing a path whose expected distance is minimum from a tree where each path to the leaf from the root results in a singular hypothesis. At various nodes of the tree the number of hypotheses is reduced as it proceeds down from the root. The nodes of the tree are computed as the best locations to move from an earlier higher hypotheses state. They are either frontier regions or a direction of traversal that result in reduction of hypotheses from an earlier multi hypotheses state. The method has been tested robustly in various simulation situations and its efficacy confirmed.

Keywords: Mobile Robots; Multi-hypotheses Localization;

1. Introduction

Among the different aspects of the navigation task, the positioning problem is the crucial one and consists of maintaining in real-time a reliable estimate of the position of one or more mobile robots with respect to a reference frame in the environment.[1] The problem of global localization –or "the kidnapped robot problem"– is that of, from little or no a priori pose information, estimating the correct pose of a mobile robot with respect to some global reference frame. This is fundamentally different from the pose tracking problem, i.e., maintaining an estimate of the robots pose as the robot moves. This is relevant not only at start up, but also during operation for recovery in case of pose tracking failure. Global localization algorithms can result in multiple hypotheses locations for a robot since the local environment seen by the robot can be seen from various other locations in a given map. In such a scenario the robot navigates to places where the local environment sensed is unique in the whole map to come up with a unique hypothesis of its position. This is commonly referred as *active localization*.

In this paper we propose a solution to the problem of actively localizing a robot that possesses multiple hypotheses of its state, where the state is defined by the tuple $< x, y >$. A localization tree is constructed as follows. The root node characterizes the set of all possible hypotheses or states of the robot, denoted by S_0. A set of actions $A_0 = \{a_{01}, a_{02}, ..., a_{0n}\}$ are enumerated for the root node. These actions typically lead to frontiers or to an obstacle that reduces the number of hypotheses. These actions embody the property that the states resulting from those actions will always have at-least one hypothesis lesser than its parent node. Links between the nodes of the tree store the distance between the hypotheses locations characterized by those nodes. From the tree that path is selected that has the minimum expected value of distance to travel before singularizing to a unique hypothesis.

The current method is different than previous popular methods of active localization in the way the next possible locations are selected and the path to the eventual location of unique hypothesis is constructed. Specifically it differs from the entropy minimization scheme of[3] in the way next possible locations are selected. By chosing frontiers or locations near objects that reduces the number of hypotheses it reduces the search space when compared with the randomly chosen points for which entropy computations are performed in.[3] Since points are randomly chosen in entropy maximization it takes more time to reach a location where the number of hypotheses gets reduced. Moreover entropy computations are more intense than computing locations of frontiers[2] and directions towards obstacles that result in a reduction of hypotheses. Unlike[4] where the path is computed only till the next node in the tree the current method provides a method for selecting a path till the leaf node there by improving the optimality of the path generated. Moreover it does not assume polygonal obstacles and infinite range sensors as assumed in.[4]

Simulation results depict the efficacy of the method and gives a vivid portrayal of the same. Comparative advantages of computing paths by constructing the complete tree vis-a-vis paths computed only till the next best node are also presented.

2. RELATED WORK

In general work on active localization has tended to be limited when compared with passive localization. The pioneering work in this area has been from[3] and.[4] In[3] a method of active localization based on maximum information gain was presented. Dudek and others[4] presented a method for minimum distance traversal for localization that works in polygonal environments without holes that they show to be NP-Hard. A randomized version of the same method was presented in.[5]

A similar problem is treated in,[8] where it uses multi-hypothesis Kalman filter based pose tracking combined with a probabilistic formulation of hypothesis correctness to generate and track Gaussian pose hypotheses online. It tracks the multiple hypotheses of a mobile robot but method is largely about the uncertainty representation and using of features in enviroment to remove disambiguity. However it does not deal with actively localizing a robot and does not provide a shepherding mechanism. In[9] a novel method of actively localizing multiple robots was presented.

3. The Methodology

The localization tree forms the basis of this section. Some aspects of the tree are discussed to facilitate easier understanding of what follows later. The root node of the tree defines the original state that encapsulates all the possible hypotheses after the first scan by the robot. Markov Localization[7] is used to compute the possible hypotheses states of the robot. Any node in the tree characterizes a set of possible hypotheses. At each node a set of possible actions are enumerated. An action at the parent node results in a node that is a level lower in the tree such that its number of hypotheses is lesser or at-most equal to the parent node. The root node is denoted by S_0. The j^{th} node at level i is denoted as S_{ij} where the first subscript refers to its level in the tree and the second subscript denotes its index where the nodes are indexed from left to right at the same level. The connotation $G(S_{ijk})$ is the set of possible states generated due to an action a_k taken at the state S_{ij}, $G(S_{0i}$ denotes the states generated due to action a_i at S_0 .

3.1. *Tree Construction*

The tree is constructed from the root node S_0 as follows. The set of all actions at S_0, $A = \{a_1, a_2, ..., a_m\}$ at S_0 is enumerated. The actions are then iteratively performed, each action generating a set of new possible states at a level lower than the root. These states are visited iteratively in a breadth first fashion, actions enumerated and performed. The whole process continues till the tree has only leaf nodes, nodes with a distinct hypothesis or there are no more new places to visit to get a unique hypothesis.

The actions could be one of moving to a frontier or reaching close to an object that reduces the number of hypotheses. For example Figure 1 shows a suite with three rooms. Action a_1 takes the robot to the frontier at the top, a_2 to the bottom and a_3 to the right close enough to detect the presence or absence of the object in room 3, the right most room. Figure 2 shows a typical localization tree of three levels. Each node in the tree is represented by its state, S_{ij}, the number of hypotheses in that state, $H_{ij} = \{H_m, H_n, ..., H_q\}$ and the set of actions for that state. Here each $H_{ij} \subseteq H_0$, H_0 the set of all hypotheses at the root node, S_0. The distance between the nodes are also shown as the weights of the links between the nodes and the computation of these distances is explained further below. $G(S_{01})$ results in nodes S_{11}, S_{12} in the Figure while $G(S_{02})$ results in S_{13} and S_{14}.

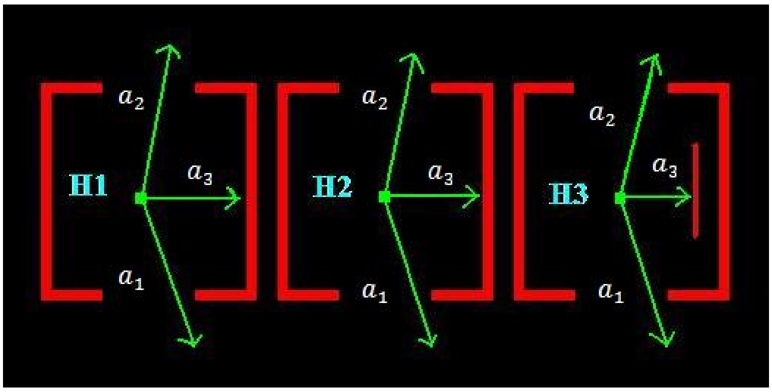

Fig. 1. A suite of three rooms and the corresponding three hypothesis. Action a_1 moves to the top frontier, a_2 to the bottom and a_3 to the right. The third action can discern H_3 from H_1 and H_2.

3.2. *Computing Distances*

The distance between two nodes S_{ij} and $S_{i'j'}$ is denoted by $d_{iji'j'}$ where $i' > i$ is at a level lower than i. The distance is computed based on the action chosen at the parent state S_{ij}. The actions are common to all the hypotheses that constitutes that state or node as shown in Figure 2. The actions are a movement to a frontier location or to a location near an obstacle that discerns some of the hypotheses of that parent node. The distances

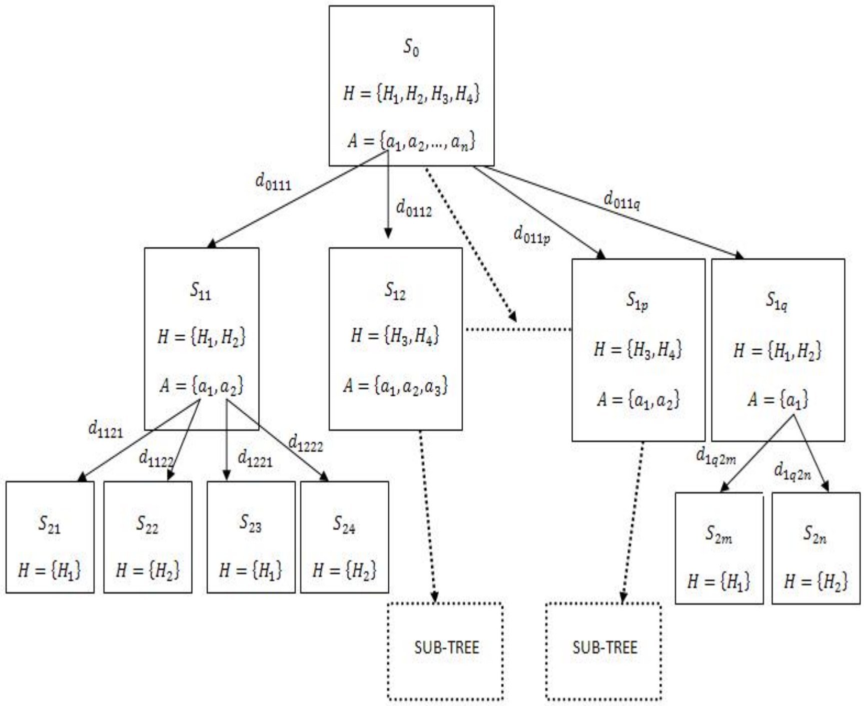

Fig. 2. Localization tree of 3 level (not all nodes are shown).

are then the distance from the hypotheses locations to the computed frontier locations or locations close to the discerning obstacle. It is worthwhile to note that the distance corresponding to a particular action is the same for all the hypotheses in the corresponding node for which the action is performed. At times these distances to frontier locations need to be recomputed invoking a path planning methodology. This happens because the frontier locations corresponding to actions at the parent state, S_{ij} were computed assuming infinite range sensor. Since the actual robot is limited by finite range it needs to move beyond the computed frontier locations to get the discerning measurement that reduces the number of hypotheses at the node $S_{i'j'}$.

Due to bervity of space we do not discuss in detail how distances between the nodes of a tree are computed due to constraints imposed by a finite range sensor.

3.3. *Best Path Computation*

Let the root node characterized by state, S_0 have the set $A = \{a_1, a_2, ..., a_m\}$ of actions associated with it. Let the initial number of hypotheses at S_0 be h_I. Then the path that has the least expected distance is given by

$$p = arg(min(E_{S_0}(a_1), E_{S_0}(a_2), ..., E_{S_0}(a_m))) \tag{1}$$

$E_{S_0}(a_j)$ represents the expected distance to be obtained by chosing action a_j from state S_0. If $G(S_{0j})$ results in nodes whose states are $S_{11}, S_{12}, ..., S_{1q}$ with corresponding distances from the root being $d_{0111}, ..., d_{011q}$ and having corresponding number of hypotheses $h_1, \ldots, h_q$ respectively, with each $h_i < h_I$, then:

$$\begin{aligned} E_{S_0}(a_j) = d_{0111}\frac{h_1}{h_I} min(E_{S_{11}}(a_1), ..., E_{S_{11}}(a_n)) + ... \\ + d_{011q}\frac{h_q}{h_I} min(E_{S_{1q}}(a_1), \ldots, E_{S_{1q}}(a_n)) \end{aligned} \tag{2}$$

Here we have assumed without loss of generality that the actions enumerated at eacj state S_{1i} to be the set $A = \{a_0, \ldots, a_n\}$. Similarly expected values of distances due to other actions at state S_0 can be found. These in turn recursively depend on the actions taken at nodes one level below in the tree which are once again computed in a manner similar to eqn(2). The recursion proceeds till the leaf nodes are reached. In eqn(2) the term $\frac{h_i}{h_I}$ denotes the probability of reaching state S_{1i} upon taking the action a_j at S_0.

4. Simulations

Figure 3 shows a map with three suites of rooms, the top and middle one are separated by a corridor while the middle suite and lower suites are adjacent to each other. The robot is in one of the rooms in the middle suite and its hypotheses locations are given by H_1, H_2, H_3 that constitutes the state S_0. It has two possible actions at S_0 one leading to the frontier at the top and the other to the bottom enumerated as actions a_1 and a_2 in Figure 4. These actions result in nodes whose states are shown as S_{11}, S_{12} due to a_1 and S_{13} due to a_2 in Figure 4. Further from state S_{11} two actions are possible, a_1 that leads to the frontier areas in the right side rooms or a_2 leading to frontier areas in the left sdie rooms. Action a_1 at S_{11} results in discerning remaining hypotheses and a similar kind of actions are possible if a_2 is chosen. Action a_1 at S_{13} moves to frontiers on the right while a_2 moves to frontiers on the left. Chosing a_2 at left results in uniquely hypothesizing the robot's state while moving to the right results in a longer tree and longer paths. The best possible paths that constitute the paths giving rise to the minimum expected value if action a_1 is chosen at S_0 is shown in green. The paths that constitute the paths in minimum expected value if action a_2 is shown in yellow. Among the two the expected value due to action a_1 at S_0 turns out lesser and hence the green paths are preferred.

Further on extensive simulations we found that building the tree to atleast level three and chosing the minimum expected path by looking three levels into the future gave rise to shorter paths on more than 20% of the times than chosing the minimum expected path greedily based on one level into the future. This vindicates the formalism developed here to choose minimum expected distance paths upto any level of the tree starting from the root.

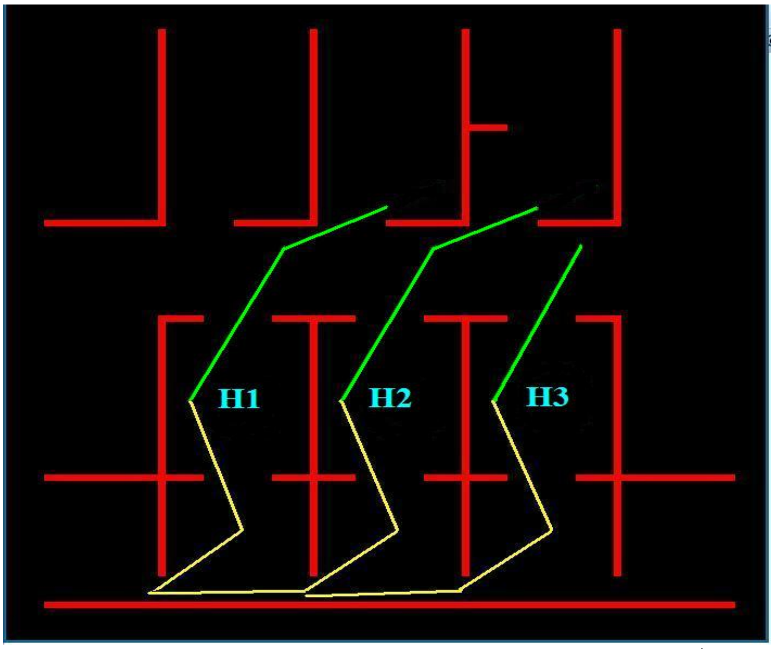

Fig. 3. The Map. The robot is in one of the three rooms of the middle suit (corresponding to the three hypothesis). The green lines indicate the optimal actions of the robot in localization. The yellow lines indicate actions where the robot has to take longer paths to localize.

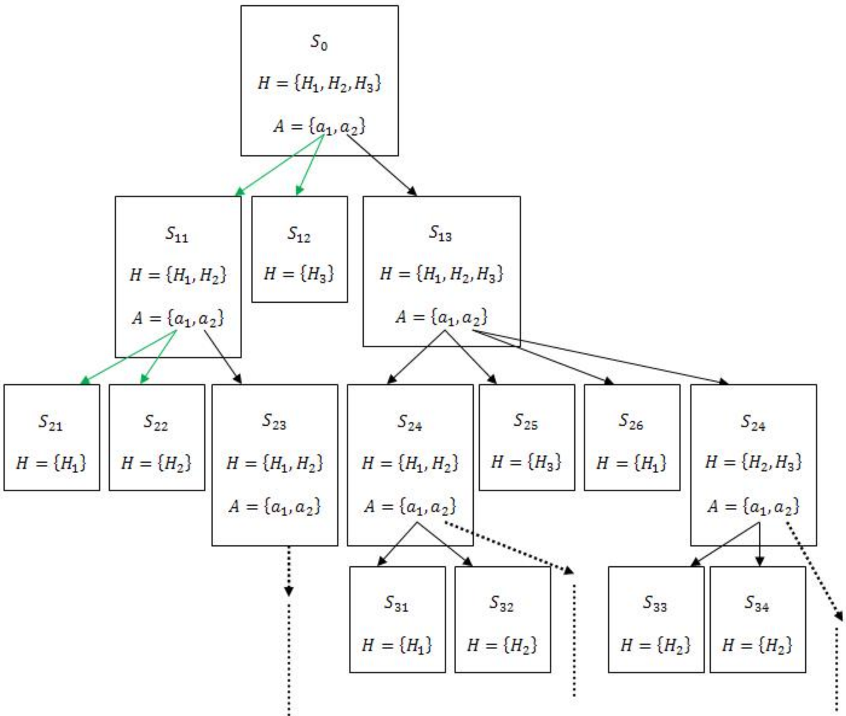

Fig. 4. Tree corresponding to the localization of robot in the map of Figure 3. Tracing the path along the green edges of the tree leads to an optimal localization of the robot.

5. Conclusions

This paper has presented a new method of computing localization paths for obtaining a unique hypothesis from a multi-hypotheses case in a global localization or kidnapped robot scenario. The various paths correspond to various traversals from the root node to the leaf node of the localization tree. That path is selected that provides for the minimum expected distance from the root to the leaf node. The method has been tested in simulations to verify its efficacy. It is different from other approaches by providing a formulation that can select the best path until any level in the tree there by enhancing its optimality and by being invariant to the geometry of the environment and sensing ranges. Further work will be to efficiently prune the tree during construction to prevent computations from becoming unwieldy.

References

1. J. Borenstein, B. Everett, and L. Feng., “Navigating Mobile Robots: Systems and Techniques” A. K. Peters, Ltd.,Wellesley, MA, 1996.

2. Yamauchi, “A Frontier-Based Approach for Autonomous Exploration”, IEEE International Symposium on Computational Intelligence in Robotics and Automation: 146-151
3. D. Fox, W. Burgard, and S. Thrun, “Active markovlocalization for mobile robots”, Robotics and Autonomous Systems, 25 (1998), 195-207
4. G. Dudek, K. Romanik, and S. Whitesides, “Localizing a robot with minimum travel”, SIAM J. Computing 27(2), 1998
5. M Rao, G Dudek, and S. Whitesides, “Minimum Distance Localization for a Robot with Limited Visibility”, Proc of ICRA, 2005
6. L. P. Kaelbing, M.L. Littman, and A.R. Cassandra, ”Planning and acting in partially observable stochastic domains”, TR, Brown University, 1995
7. Fox et. al, ”A Probabilistic Approach to Collaborative Multirobotic Localization”, Autonomous Robots, 8(3), 2000
8. Patric Jensfelt and Steen Kristensen, “Active Global Localization for a Mobile Robot Using Multiple Hypothesis Tracking”, IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, VOL. 17, NO. 5, OCTOBER 2001
9. S Bhuvanagiri, K Madhava Krishna and R Goyal, “Active Localization of Multiple Robots by Moving to Best Frontiers”, International Conference on Advanced Robotics - ICAR’07, Jeju, Koreadh