Active Global Localization for Multiple Robots by Disambiguating Multiple Hypotheses

Shivudu Bhuvanagiri and K Madhava Krishna

***Abstract*— In environments which possess relatively few features that enable a robot to unambiguously determine its location, global localization algorithms can result in multiple hypotheses locations of a robot. In such a scenario the robot, for effective localization, has to be actively guided to those locations where there is a maximum chance of eliminating most of the ambiguous states – which is often referred to as 'active localization'. When extended to multi robotic scenarios where all robots possess more than one hypothesis of their position, there is the opportunity to do better by using robots apart from obstacles as 'hypotheses resolving agents'. The paper presents a unified framework accounting for the map structure as well as measurement amongst robots while guiding a set of robots to locations where they can singularize to a unique state. The appropriateness of our approach is demonstrated empirically in both simulation & real-time (on Amigobots) and its efficacy verified. Extensive comparative analysis portrays the advantage of the current method over others that do not perform active localization in a multi-robotic sense.**

I. INTRODUCTION

Among various aspects of the navigation task, the positioning problem is crucial and consists of maintaining in real-time a reliable estimate of the position of robots with respect to a reference frame in the environment [1]. The problem of global localization is that of, from little or no a priori pose information, estimating the correct pose of a mobile robot with respect to some global reference frame. In symmetric environments, local map could look the same from multiple positions. So, global localization algorithms result in multiple hypotheses of robot location. In such a scenario the robot navigates to places where the local environment sensed is unique in the whole map to come up with a unique hypothesis of its position. This is commonly referred as *active localization*.

This paper presents a new approach to the problem of *actively localizing* a group of mobile robots, where all robots possess more than one hypothesis of their position, capable of sensing one another. The proposed method moves robots with multiple hypotheses to places such that a maximum number of such robots singularize to a unique state. We consider frontier locations [2] as good places to move to localize since they are easy to compute and provide a sufficient set of places to visit for convergence to a unique hypothesis. The method presented considers the best frontiers to move for a set of robots such that the probability of finding a unique hypothesis for the set is maximum. Intuitively the best frontiers are those upon robots reaching them, gives rise to a maximum number of unique measurements between robots (robot-robot detections) as well as from robots to the local map structure at those places. It is worth noting that multiple hypotheses arise because there is no unique measurement at those hypotheses locations that is able to discern that location from the other competing hypotheses locations. Hence the search for frontiers that provide as many unique measurerments as possible. Since the initial states of the robots are multiple a robot cannot hypothesize to reach any frontier location uniquely but can only in a probabilistic sense. Hence a probabilistic framework is developed that finds the probability of reaching to places where unique measurements leading to unique hypotheses is maximum for a set of robots. Once shepherded to these positions in a probabilistic sense some of the robots could attain a unique hypothesis. This framework also accounts for other constraints such as the detecting range of the sensors and the local map structure at the frontier locations detailed in Section III.

Extensive comparative analysis portrays the advantage of the current method over others that do not perform active localization in a multi-robotic sense. It also delineates the performance gain by considering map structure and robot placement to actively localize over methods that consider only one of them or neither.

The authors argue that the novelty of this work lies in that it is the first such method where multiple hypotheses states in more than one robot is resolved in an active localization sense. The essential contribution apart from the method itself is the clear delineation that performing active localization with multiple robots is advantageous than active localization performed singularly when other robots are present.

The method finds application when robots operate in symmetric environments with few distinguishable features. These can occur indoors in corridors with symmetric rooms or evenly spaced pillars, or outdoors in large open spaces scattered with trees that are far apart. The essential motivation has been to develop a framework for active localization with multiple robots – to analyse the performance gain over a framework where active localization is done individually.

II. RELATED WORK

In general work on active localization has tended to be limited when compared with passive localization. The pioneering work in this area has been from [3] and [4]. In [3] a method of active localization based on maximum information gain was presented. Dudek and others [4] presented a method for minimum distance traversal for localization that works in polygonal environments without holes that they show to be

Robotics Research Center, International Institute of Informations Technology, India - 500032. shivudu@research.iiit.ac.in, mkrishna@iiit.ac.in

NP-Hard. A randomized version of the same method was presented in [5]. In [6] an approach for guiding the robot to a target location is proposed when its current position is not known accurately.

A similar problem is treated in [12], where it uses multi-hypothesis Kalman filter based pose tracking combined with a probabilistic formulation of hypothesis correctness to generate and track Gaussian pose hypotheses on-line. It tracks the multiple hypotheses of a mobile robot but method is largely about the uncertainty representation and using of features in enviroment to remove disambiguity. However it does not deal with actively localizing multiple robots and does not provide a shepherding mechanism.

As far as authors knowledge goes there has been no reported work dealing active localization of several robots. In the context of multi robotic localization Fox et.al. extended their earlier global localization methods to a multirobotic setting in [7]. A coordinated localization approach was also presented based on Monte-Carlo methods in [8]. In [9] a method is presented in which team members carefully coordinate their activities in order to reduce cumulative odometric errors. There has been a class of methods that make use of relative observations between robots to reduce absolute position errors by optimizing a objective function such as in [10] or through a sensor fusion formulation using EKF [11]. Vision-based relative localization is used for maintaining formation in [14]. The robots however profess only a singular hypothesis while the uncertainty in their hypothesis is reduced through relative observations. The above methods do not consider active localization in a definite sense of guiding robots to positions from where they can obtain a unique hypothesis of their states from an initial multi modal situation. Hence they cannot be classified as active methods of localization while they certainly deal with multiple robots.

III. ACTIVE LOCALIZATION

Consider a workspace W populated by robots, n_R in number, each of them having multiple hypotheses of their states. The problem is to move these n_R robots to frontier locations such that each of the robots localizes to a unique state with minimal number of frontiers visited. By a state or hypothesis we imply a position $\langle x, y \rangle$ in a map. We assume the robot's orientation is known through a compass.

The problem is explained in the following fashion: Firstly the computation of probability of robots reaching or occupying frontiers (occupancy probability) is depicted. Secondly the probability of obtaining at-least one unique measurement, also denoted as UM, for a set of robots is shown. This probability arises due to both the nature of local map structure and the placement of other robots. Both the cases are discussed. Also the modifications in the probability due to sensing range limits and other visibility constraints is also presented. Thirdly how the probability of a unique hypothesis is computed from UM probabilities is depicted. Lastly we show how the computations can be reduced by clustering robots into what are called as *base-pairs*.

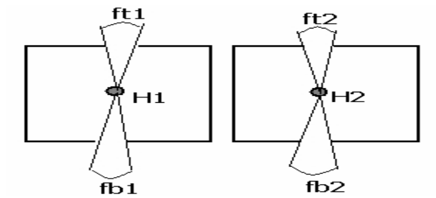


Fig. 1. Robot with two hypotheses states H1 & H2. There are two virtual frontiers (ft1, ft2) for every distinct frontier due to two hypotheses states.

A. Hypotheses and Frontiers

The input for the problem being addressed is *multiple robots with multiple hypotheses of their state*. We compute hypotheses from the multi-modal belief of pose in Markov global localization.

The notion of a *distinct frontier* is used while specifying the direction to move the robot. The direction is always to one of the surrounding frontiers from a hypothesis location. These directions are distinct since the frontiers are distinctly oreinted with respect to a robot hypothesis.

However since there are multiple hypotheses (say n_h in number), the robot reaches to one among the n_h frontier locations while moving along a direction. Despite selecting a unique direction, it reaches to one of the n_h possible *virtual frontiers*. Hence forth when a robot move to a frontier it refers to one among the various virtual frontiers, the distinct frontier is used to refer to the direction choosen.

B. Computing Occupancy Probabilities

Imagine a map where there are n such similar rooms as in Figure 1. Then a globally localizing robot would have n hypotheses. If it decides to occupy the distinct frontier in the top it would end up reaching one of those n virtual copies in $1/n$ ways. More formally if there is a set of robots R with cardinality n_R capable of occupying a set of frontiers F, with cardinality n_F, the probability a particular k of those, say $f_1,\ f_2,\ \cdots,\ f_k$ of them gets occupied is given by

$$\begin{aligned} P_o(n_F, n_R, k) &= \frac{{}^{n_R}P_k \times {}^{(n_F-k)}P_{(n_R-k)}}{{}^{n_F}P_{n_R}}; n_R \geq k \\ &= 0; n_R < k \end{aligned} \quad (1)$$

Here the term $P_o(n_F, n_R, k)$ denotes the probability of occupancy of a particular k of the n_F frontiers by n_R robots. The first term in the parenthesis denotes the number of frontiers, the second the number of robots and the third number of frontiers to be occupied. The notation ${}^xP_y$ denotes the permutation of x things taken y at a time.

C. Computing Unique Measurement Probabilities

Let there be a set R of n_R robots, $r_1, r_2, ..., r_{n_R}$. Let each robot $r_i \epsilon R$ have n_d distinct frontiers to move to, each distinct frontier having n_f virtual frontier copies. The objective is to come up with the right combination of robots moving to distinct frontiers such that the probability of obtaining a unique measurement is a maximum for the entire set. The unique measurements discern a frontier location from the remaining $n_f - 1$ for a robot and results in a unique hypothesis. The unique measurements are of two kinds. They

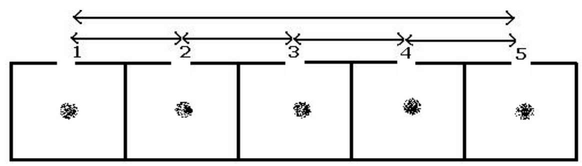


Fig. 2. A suite of 5 identical rooms with an opening on top. Occupancy of the extreme frontiers by robots gives rise to a UM (longest arrow on top).

are measurements between a pair of robots from frontier locations that cannot be replicated from any other pair of frontier locations that can be reached by those robots. Unique measurements are also due to the local map structure at one or more of the n_f frontiers of a robot r_i that does not occur in any of the remaining frontiers.

1) Unique Measurements Between Robots: A measurement between two frontiers is denoted by the tuple $\langle d, \theta \rangle$ and measures the distance and angle between the two. A unique measurement between a pair of frontiers is a tuple that is not measured between any other pairs. For example in a symmetric five room suite in Figure 2 the initial global localization algorithm gives rise to five possible hypotheses for each of the robots, say two, in those rooms. The only pair of unique measurement between frontiers is when the end frontiers at the top get occupied (Figure 2) since the frontier locations are equally spaced. The probability of occupancy of the two end frontiers from the two robots is the computation of $P_o(5, 2, 2)$. For measurements between any other pair of frontiers get replicated elsewhere. For example measurement between first and second frontiers is the same as between 2 and 3, 3 and 4 & 4 and 5 since they are spaced equally.

Define an allocation of robots to distinct frontiers as $alloc_i = (r_1 \rightarrow f_1, r_2 \rightarrow f_k, ..., r_{n_R} \rightarrow f_j)$. The above allocation moves robot r_1 to frontier f_1, r_2 to f_k, ..., r_{n_R} to f_j. Denote by UM^i_{alloc} the set that contains all the possible pairs of virtual frontiers that result in a unique measurement between them due to the allocation $alloc_i$. For example in Figure 2 for the suite of 5 rooms let the distance between frontiers 2 and 3 be *different* while all the distances between adjacent frontiers namely 1,2; 3,4; 4,5 be *same*. Then the following pairs of frontiers gives rise to unique measurements between them namely 1,5; 2,3; & 3,5. They would belong to the set UM^i_{alloc} due to the allocation $alloc_i$ that sends all the robots in those five rooms to the top distinct frontier. Denote by $P_{UM}(R-R, alloc_i)$ the probability of obtaining at-least one unique measurement due to robot-robot detections (measurements made between robots) through allocation, $alloc_i$. It is computed as

$$P_{UM}(R-R, alloc_i) = \bigcup_{p \epsilon UM^i_{alloc}} P_{occupancy}(p) \qquad (2)$$

The rightmost term $P_{occupancy}(p)$ refers to the probability of occupancy of the pair of frontiers $p \epsilon UM^i_{alloc}$ and is of the form $P_o(n_F, n_R, 2)$ computed through (1). The above equation merely finds the union of occupancy probabilities of all those pair of frontiers that gives rise to a unique measurement due to robot-robot detections between those frontiers.

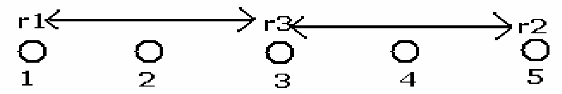


Fig. 3. Five equally spaced frontiers (circles). Though r3 is present between them, r1 and r2 can localize to a unique state by mediating via r3.

When robots reach their alloted frontiers to detect one another they may not due to presence of obstacles or due to limited range of sensors. The probability of obtaining a unique measurement then needs to account for these situations and its done as follows.

D. Incorporating Visibility Constraints

While computing the probability of obtaining a unique measurement (UM) the implicit assumption has been that the robots occupying the frontier pair are visible to one another. Incorporating visibility constraints takes into considerations the situations when there are obstacles between the frontier pair and cases where they are not within sensing range of each other. Figure 4 captures a situation when UM cannot be realized because of obstacle, but the same can realized looking through other robots. Belief update between two robots is done according to the Markov framework of [3]. Similarly, in figure 3 when the UM frontiers 1 & 5 are not in sensing range we make use of robot, r_3, present at intermediate frontier 3. In these cases, computing probability of obtaining a particular UM needs more than two robots to occupy the frontiers. How multiple robots update their beliefs between each other is not discussed here and we cite [13] for this purpose.

E. Unique Measurement due to Map Structure

The local map structure around the considered frontier can also provide a discerning measurement that discerns that frontier from all others. For example Figure 5 contains four room suite with a robot inside having four hypotheses locations. If the robot manages to reach the two right most virtual frontiers at the top it will obtain at-least one unique measurement due to the local map, however if it reaches the bottom frontiers there are no unique measurements. Then the probability of obtaining at-least one unique measurement if it moves to the top is the probability of occupying the two right most frontiers and is given by $P_o(4,1,1) \cup P_o(4,1,1)$. Let the set $UM^i_{alloc,Map}$ be the set of all frontier locations that make a unique measurement onto the map due to allocation, $alloc_i$. Then the probability of at-least one unique measurement

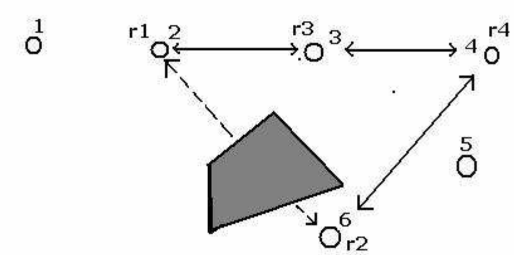


Fig. 4. Though robots r1 and r2 at 2 and 6 cannot detect one another they still could localize to a unique hypothesis if r3 and r4 occupy two of the frontiers labeled 3, 4 & 5.

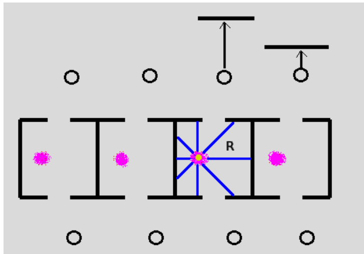


Fig. 5. shows four hypotheses for robot R with priors as pink clusters. Only the two right frontiers for the distinct frontier at the top can give a unique measurement due to the local map structure there.

through the allocation, $alloc_i$, from the local map is given by

$$P_{UM}(Map, alloc_i) = \bigcup_{m \epsilon UM^i_{alloc,Map}} P_{occupancy}(m) \quad (3)$$

The rightmost term $P_{occupancy}(m)$ refers to the probability of occupancy of the frontiers $m\epsilon UM^i_{alloc,Map}$ and this is of the form $P_o(n_F, n_R, 1)$ computed through (1). The above equation merely finds the union of occupancy probabilities of all those pair of frontiers that gives rise to a unique measurement due to measurements made on the local map structure at those frontiers.

F. Choosing the Best Allocation

The overall probability of obtaining a unique hypothesis due to an allocation is given by

$$P_{UH}(alloc_i) = P_{UM}(R - R, alloc_i) \cup P_{UM}(Map, alloc_i) \quad (4)$$

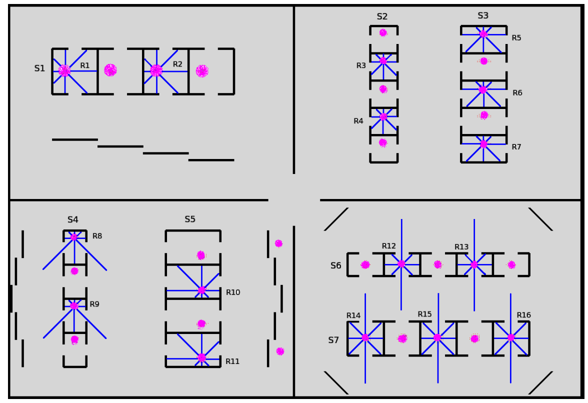
The best allocation is then chosen as

$$alloc_{best} = arg(\max_j P_{UH}(alloc_j)) \quad (5)$$

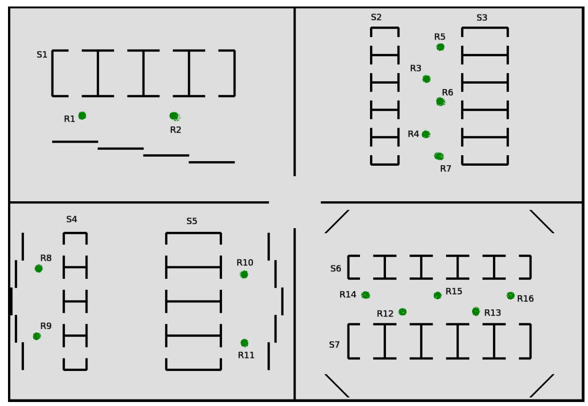
When more than one allocation has the same probability of obtaining a unique hypothesis that allocation is chosen that has the highest probability of eliminating a maximum number of hypotheses.

G. Clustering to Form Base-Pairs

Since the number of combinations of robots with frontiers is exponential in the number of robots, the original set of robots R is now partitioned into N mutually exclusive subsets R_1, R_2, $\cdots$, R_N where the cardinality of set R_i is n_{R_i} and $\sum_i n_{R_i} = n_R$. The partition is done in a manner such that all robots in the partition share the same set of hypotheses and frontiers. By sharing we mean for every virtual frontier f_{vi} for robot $r_m \in R_i$ there exists exactly one virtual frontier f_{vj} for every other robot $r_n \in R_i$, $r_m \neq r_n$ that is close to f_{vi} in a Euclidean sense, their distance less than a threshold. If such frontiers are not found among other robots then r_m is the only member of R_i. We call each such partition of robots as a base pair – denoting the pairing of the set of frontiers with the set of hypotheses. The number of base pairs equals the number of mutually exclusive subsets. We denote base pair i as fr_i, f indicates the frontiers shared by the robots in that base pair. The partitioning reduces the number of combinations to be considered while moving to best frontier locations.



(a) Robots with Multiple States.



(b) All robots are localized.

Fig. 6. Snapshots of simulation demonstrating active localization.

The equations 1, 2 and 3 are then dovetailed to address allocations of base pair robots to frontiers than individual robots to frontiers. In other words $alloc_i$ is now defined as $alloc_i = (R_1 \rightarrow f_1, R_2 \rightarrow f_k, ..., R_N \rightarrow f_j)$, i.e. allocation of a robot set R_i of the base pair fr_i to a frontier than allocation of individual robots to frontiers. The equations are then read with this modified understanding of an allocation. This is possible because the robots in a base pair are indistinguishable in behaviour and hence would need same action. Hence allocation can be talked in terms of the whole set of robots than individual robots.

IV. Simulation Results

Figure 6(a) shows the map with 16 robots arranged in 7 suites named S1, S2, $\ldots$, S7. The robots are labeled R1, R2, $\ldots$, R16 which are placed as shown and the blue lines show the initial scan of the sensors obtaining mutli-modal probability distribution for the 16 robots. The hypotheses positions thus computed as the expectation of the prior is shown as pink cluster (with a mean). Robots in suites S1, S6 & S7 have two distinct frontiers - one at top and one at the bottom, while those in the remaining suites have distinct frontiers to left and right.

Figure 6(b) captures the situation after the robots have moved to the frontiers choosen by best allocation given by equation (5). The robots in S1 are moved to bottom

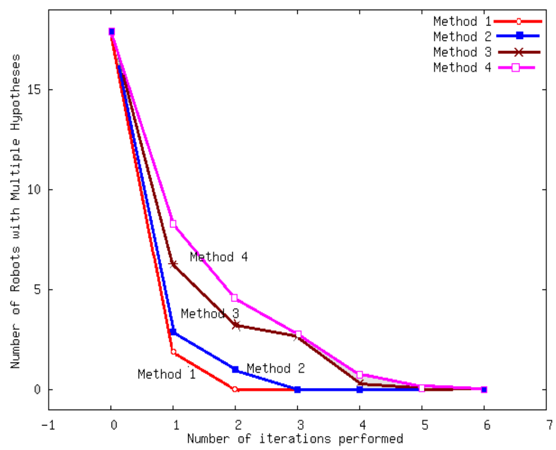


Fig. 7. Graph showing number of iterations versus number of robots with multiple hypotheses (robots remaining to be localized) for a fixed number of robots, 18, averaged over different maps.

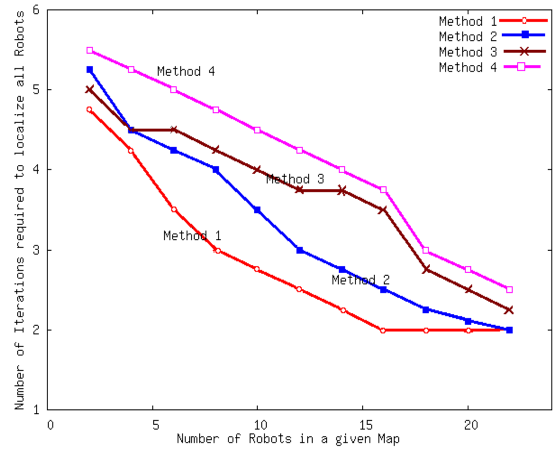


Fig. 8. Graph showing number of robots in a given map versus number of iterations for each method to localize all the robots in that map (averaged over a large no: of maps).

frontiers because the local map structure provides more unique measurements at the bottom than at the top frontiers. The unique measurements due to robot-robot detection are same either at top or bottom frontiers. Thus in computing (4) moving bottom has a higher probability due to predominance of the term due to the map given by (3). The robots in suites S2 & S3 are guided to frontiers on right & left respectively as (4) evaluates high due to predominance of probability due to robot-robot detections (2) on doing so. Robots in S4 & S5 have more unique measurements due to robot-robot detections if they move towards right & left whereas moving left & right results in more unique measurements due to local map structure. Since the probability due to equation 3 is more than that by equation 2 the robots from S4 move to left and robots in S5 to their right. In the case of suites S6 & S7 the probability given by equation 2 dominates the probability due to equation 3 by allocating robots in S6 to bottom frontiers and the robots in S7 to their top frontiers. This is due to higher probability of localizing due to unique measurements between robots than by local map structure.

It is to be noted that robots start from exactly the same relative positions so as to demonstrate the basepair clustering. Otherwise robots could start from any position resulting in more robot-frontier combinations (more basepairs for same number of robots) to be considered.

A. Comparative Analysis

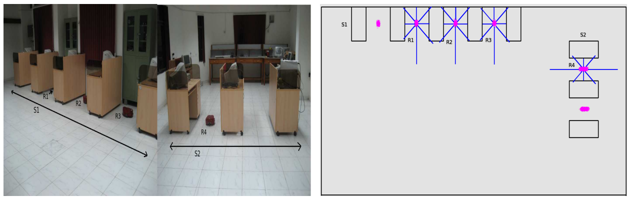
Here we compare the current method (Method 1 in the graphs) with three other methods of active localization. The second method (Method 2) actively localizes by computing the best allocation based on unique measurements between robots alone and not due to the map structure. The third method (Method 3) computes the best allocation based on unique measurements due to the local map structure neglecting measurements between robots. The fourth method decides upon an allocation randomly. All the methods upon reaching their alloted frontiers do take measurements on the robots and the local map to localize faster. Its only while deciding the allocation some of these measurements are not considered. Graphs of Figures 7 and 8 correspond to the first set of comparison. It essentially shows the performance gain of taking into account the placement of robots and the local map structure together over methods that take into account only one or neither.

Figure 7 plots the number of frontiers visited or equivalently the number of iterations of the algorithm on the x-axis and the number of robots with more than one hypothesis on the ordinate. Method 1 (orange & circles), the current method localizes all the robots to a unique state with the least number of frontiers visited. Method 4 (pink & open squares) that allots frontiers randomly visits the highest number of frontiers to localize all and comes last. Method 2 (blue & filled squares) that considers only robot measurements in deciding the allocation perfoms the second best while method 3 (brown & crosses) that considers only local map structure performs third best. Graphs are the results of simulations performed over several maps and averaged over several runs, each map having the same number of robots, eighteen.

Figure 8 captures the trend of increasing number of robots visiting the same area across methods. It plots number of robots on the abscissa and number of iterations of the algorithm or number of frontiers to visit to localize all the robots on the ordinate. As the number of robots increase the number of iterations to localize all the robots decreases across all the methods as all of them do consider detections between robots at the time of localizing their beliefs. The current method takes the least number of iterations to localize across all number of robots considered (2 to 24). The method that allots randomly (Method 4) takes the maximum number of iterations to localize. Method 3 is faster than method 2 when number of robots is lesser since the map measurements dominate, while 2 is faster than 3 when number of robots are more since measurements between robots become more prominent. Fractional number of iterations on the ordinate is a result of average taken over a large number of runs for every robot considered on the abscissa, i.e., $\{2, 4, 6, \cdots, 24\}$.

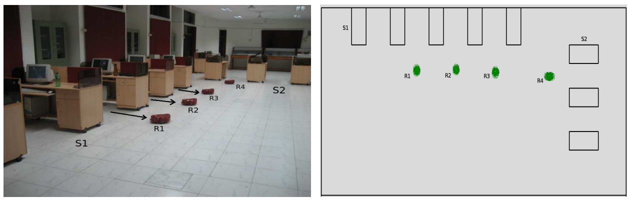
V. IMPLEMENTATION

The method was verified on a pack of Amigobots equipped with 8 sonar transducers. External hardware in the form of



(a) Real Setup with 4 Amigos. (b) Simulator Counterpart.

Fig. 9. Setup for the real-time implementation of Active Localization.



(a) On moving to frontiers. (b) Simulation counterpart.

Fig. 10. Snapshots after active localization algorithm execution.

IR transceiver circuit was interfaced to the serial buffer of the Amigobot's controller board to facilitate easy detection of one robot by other. Each transceiver transmitted a unique pulse code corresponding to a particular robot. It also received similar codes corresponding to other robots. The hardware apart from detecting a robot was capable of measuring the bearing between the detected and detector robot. Sonar sensors are then fired along the detected direction to measure distance between the two.

Figure 9(a) shows a placement of work desks in our lab that is naturally symmetric. It contains two orthogonal columns, the longer containing five desks and the smaller three. These are referred as suites S1, S2 in figure 9(b), that captures the same lab map in a graphics simulator as in section IV. There are three Amigos in S1 and one in S2. For those in S1 there is only one distinct frontier to the bottom and for those in S2 there are two distinct frontiers one to the left and other to the right. There are four virtual frontiers for the only distinct frontier corresponding to four hypothesis for each Amigo in S1. There are two virtual frontiers for each distinct frontier for the Amigo in S2. Amigos in S1 and their distinct frontiers form one base-pair and the Amigo in S2 forms the other. The best allocation is one that moves Amigos in S1 down and the Amigo in S2 to left to maximize their chances of attaining a unique hypothesis. Figure 10(a) shows the situation when the Amigos have reached their alloted frontiers. Figure 10(b) shows the simulator counterpart with the robots having localized to a unique hypothesis, posterior distributions in green.

VI. CONCLUSION

The paper is about guiding several robots who are in ambiguity of their states to locations where as many of them can get rid of their ambiguities by localizing to a unique hypothesis state. It presents a unified probabilistic framework that takes into account the role of measurements between robots as well as the measurement made on the local map structure in deciding the best locations to move. It also tackles within this framework the constraints due to sensing range and the presence of obstacles that prevent robots from detecting each other by a unique measurement leading to a unique hypothesis. The algorithm shepherds robots to those locations where probability of localizing the robots to a unique hypothesis is maximum.

Simulation results and real experiments confirm the efficacy of the method. Extensive comparisons establish the performance gain of the current method that considers both local map structure and placement of robots in deciding where to move over those methods that consider only one of them or neither. It also clearly delineates that performing active localization with multiple robots is always advantageous than active localization performed singularly when other robots are present.

This method finds utility in several multi-robotic scenarios, where robots are not clear about their state, have multiple hypotheses and require the assistance of other robots to resolve conflicts as well as to refine their states to precise coordinates.

The future work would address the active localization with multiple robots when the robot state includes orientation and is represented by the tuple $< x, y, \theta >$. The role for multi robot active localization is expected to be enhanced in such scenarios since the inclusion of orientation increases the possibility of multiple hypotheses dramatically. Our immediate focus is on devloping a decentralized mechanism for the proposed framework.

REFERENCES

[1] J. Borenstein, B. Everett, and L. Feng., "Navigating Mobile Robots: Systems and Techniques" A. K. Peters, Ltd.,Wellesley, MA, 1996.

[2] Yamauchi, "A Frontier-Based Approach for Autonomous Exploration", IEEE International Symposium on Computational Intelligence in Robotics and Automation: 146-151

[3] D. Fox, W. Burgard, and S. Thrun, "Active markovlocalization for mobile robots", Robotics and Autonomous Systems, 25 (1998), 195-207

[4] G. Dudek, K. Romanik, and S. Whitesides, "Localizing a robot with minimum travel", SIAM J. Computing 27(2), 1998

[5] M Rao, G Dudek, and S. Whitesides, "Minimum Distance Localization for a Robot with Limited Visibility", Proc of ICRA, 2005

[6] L. P. Kaelbing, M.L. Littman, and A.R. Cassandra, "Planning and acting in partially observable stochastic domains", TR, Brown University, 1995

[7] Fox et. al, "A Probabilistic Approach to Collaborative Multirobotic Localization", Autonomous Robots, 8(3), 2000

[8] A Howard, M J Mataric and G S Sukhatme, "Putting the I in the team.: Ego-centric approach to multi-robotic localization" IEEE ICRA, 2003

[9] I. M. Rekleitis, G. Dudek, and E. Milios, "Multi-robot exploration of an unknown environment: efficiently reducing the odometry error", 2: 13401345, IJCAI 1997

[10] J R Spletzer and C J Taylor, "A Bounded Uncertainty Approach to Multi-Robot Localization", IROS 2003, pages 1258-1267

[11] A Martinelli, F Pont and R Siegwart, "Multi-Robot Localization Using Relative Observations" ICRA 2005, pages 2808-2813.

[12] Patric Jensfelt and Steen Kristensen, "Active Global Localization for a Mobile Robot Using Multiple Hypothesis Tracking", IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, VOL. 17, NO. 5, OCTOBER 2001

[13] Valguima Odakura and Anna H. R. Costa, "Cooperative multi-robot localization: using communication to reduce localization error", Automation and Robotics - ICINCO'05, pages 88-93

[14] J. Spletzer, A. K. Das, R. Fierro, C. J. Taylor,V. Kumar, and J. P. Ostrowski, "Cooperative Localization and Control for Multi-Robot Manipulation", IROS 2001