

OPTIMAL POSTURE CONTROL FOR FORCE ACTUATOR BASED ARTICULATED SUSPENSION VEHICLE FOR ROUGH TERRAIN MOBILITY

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In this paper we develop a model of a Linear Force Actuator based actively articulated suspension vehicle with toroidal wheels and propose a strategy to control its contact forces to improve traction on uneven terrain in 3-D while maintaining a desired posture. We develop the quasi-static analysis for our vehicle and compare the maneuverability of our vehicle with that of a passive suspension system. Extensive uneven terrain simulations depict the efficacy of our proposed system.

1. INTRODUCTION

To improve the mobility of wheeled robots traversing on and uneven terrain having slopes in all three orthogonal directions is the primary focus of our research. Past research on ‘all terrain vehicles’, [1], [2], was focused on developing mechanical suspension systems which could improve terrain adaptability and locomotion. Consequently control algorithms were developed for posture stability of the vehicle [3], [4]. Shrimp robots [1] and Rocky rovers [2] are terrain vehicles with passive suspension systems which have excellent terrain adaptability and ability to negotiate terrains having discontinuities that are higher than the wheel radius. But mobility and stability of such vehicles is not guaranteed. Thus for such conditions Sreenivasan and Waldron [4] developed vehicles called Wheeled and Actively Articulated Vehicles

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(WAAVs). Ch. Grand *et al* [5], [8] developed another type of such vehicle called Hybrid Wheel Legged vehicle (Hylos). Posture control algorithm for Hylos was developed by mapping the velocities at various joints to the velocity of the main body based on posture error which improves traction and stability. K. Iagnemma and S. Dubowsky [6] developed a traction control algorithm to improve ground traction of a vehicle traversing on rough terrain while optimizing power consumption. In our research we propose a Linear Force Actuator Vehicle (LFA-V) which can be characterized as a WAAV type of system to control the contact forces at the wheels while maintaining desired posture. We also introduce a method for calculating the traction force at each wheel and present the Quasi Static Analysis for the system in a 3D reference frame. The depiction of the enhanced feasibility regions of LFA-V compared with that of passive suspension vehicle confirm the efficacy of the proposed method.

2. ANALYSIS OF PASSIVE SUSPENSION SYSTEM

In this section we analyze a passive suspension vehicle on an uneven terrain and introduce our motivation to develop a force actuator based suspension vehicle.

Let ψ , ro and yo be the pitch, roll and yaw angles respectively of the chassis about the global $\{XYZ\}$ axes respectively. The resultant rotation matrix relating position vectors is given by [7]

$$R = R_z(yo).R_y(ro).R_x(\psi) \quad (1)$$

Where $R_x(\psi)$, $R_y(ro)$ and $R_z(yo)$ are the rotation matrices corresponding to the Euler angles about the X , Y and Z axes respectively. For this system two forces act at the point of contact. Normal force $\bar{N}_i = [N_{xi} \ N_{yi} \ N_{zi}]^T$ and Traction force $\bar{T}_i = [T_{xi} \ T_{yi} \ T_{zi}]^T$. Since the motion of our vehicle is in the YZ plane, there is no loss of generality in assuming

$$T_{xi} \approx 0 \text{ and } T_{yi} > 0 \quad (2)$$

Under no slip conditions we have

$$|\bar{T}_i| \leq \mu_i |\bar{N}_i| \quad \forall i = \{1,2,3,4\} \quad (3)$$

Where μ_i is the coefficient of friction between the point of contact of i^{th} wheel and the terrain. Since \bar{T}_i is always perpendicular to \bar{N}_i . We have

$$\text{dot}(\bar{T}_i, \bar{N}_i) = 0 \Rightarrow N_{xi}T_{xi} + N_{yi}T_{yi} + N_{zi}T_{zi} = 0 \quad (4)$$

From (3) the maximum magnitude of $|\bar{T}_i|$ under no slip condition is $\mu_i |\bar{N}_i|$. i.e

$$\left| \bar{T}_i \right| = \mu_i \left| \bar{N}_i \right| \Rightarrow T_{xi}^2 + T_{yi}^2 + T_{zi}^2 = \mu_i^2 N_i^2 \quad (5)$$

Case 1: If $\left| \bar{N}_i \right| = 0$ then $\left| \bar{T}_i \right| = 0$. Case 2: If $\left| \bar{N}_i \right| \neq 0$ then any one of the components $N_{xi}, N_{yi}, N_{zi} \neq 0$, let $N_{zi} \neq 0$, then from (4) and (5) we get

$$\mu_i^2 \left| \bar{N}_i \right|^2 - T_{xi}^2 = T_{yi}^2 + \left[\frac{N_{xi} T_{xi} + N_{yi} T_{yi}}{N_{zi}} \right]^2 \Rightarrow a T_{yi}^2 + b T_{zi} + c = 0 \quad (6)$$

Where $a = (N_{zi}^2 + N_{yi}^2)$, $b = 2.N_{xi} T_{xi} N_{yi}$, $c = T_{xi}^2 (N_{zi}^2 + N_{xi}^2) - \mu_i^2 \left| \bar{N}_i \right|^2 N_{zi}^2$

.Since $a \neq 0$ equation (6) is quadratic in nature and will have real roots if

$$b^2 - 4ac \geq 0 \Leftrightarrow \frac{\mu_i \left| \bar{N}_i \right|}{\left| N_{zi} \right|} \geq T_{xi} \text{ which is always true by (2). Therefore we get}$$

$$T_{yi} = \frac{\mu_i \left| \bar{N}_i \right| \left| N_{zi} \right|}{\sqrt{N_{zi}^2 + N_{yi}^2}} \quad \text{and} \quad T_{zi} = \frac{-\mu_i N_{yi} \left| \bar{N}_i \right| \left| N_{zi} \right|}{N_{zi} \sqrt{N_{zi}^2 + N_{yi}^2}} \quad . \quad \text{Now let}$$

$$\hat{t}_i = \frac{\bar{T}_i}{\left| \bar{T}_i \right|} = [t_{xi} \ t_{yi} \ t_{zi}]^T \quad \text{and} \quad \hat{n}_i = \frac{\bar{N}_i}{\left| \bar{N}_i \right|} = [n_{xi} \ n_{yi} \ n_{zi}]^T \quad . \text{Now quasi-static}$$

analysis is done on the system assuming the masses of legs and wheels are negligible when compared to the mass of the chassis. Net force and net moment acting on the system is given

$$\text{by } \bar{F} = [F_x \ F_y \ F_z]^T = \sum_{i=1}^4 (\bar{T}_i + \bar{N}_i) + M\bar{g} \quad . \quad M \text{ is the mass of the chassis and } g \text{ is}$$

the acceleration due to gravity and

$$\bar{M} = \sum_{i=1}^4 [\bar{r}_{mi} \times (\bar{T}_i + \bar{N}_i)] = [M_x \ M_y \ M_z]^T \quad . \quad \bar{r}_{mi} \text{ is the radius vector from}$$

the CG of the chassis to the point of contact of the i^{th} wheel. $2a \Rightarrow$ Length of the chassis, $2w \Rightarrow$ Width of the chassis

$$\bar{r}_{mi} = \bar{r}_{fai} + \bar{r}_1 + \bar{r}_2 + \bar{r}_3 \quad \text{where, } \bar{r}_1 = R.[0 \ 0 \ -l_i]^T, \bar{r}_3 = -r_t \cdot \hat{n}_i.$$

$$\bar{r}_{fai} = \begin{cases} R.[-w \ (-1)^{i+1} a \ 0]^T \quad \forall i \in \{1,2\} \\ R.[w \ (-1)^{i+1} a \ 0]^T \quad \forall i \in \{3,4\} \end{cases}$$

$$\bar{r}_2 = R.[0 \ -r \sin(\gamma_i) \ -r \cos(\gamma_i)]^T, \gamma_i \approx \arctan\left(\frac{N_{yi}}{N_{zi}}\right)$$

Where l_i is the length of the i^{th} leg, r is the radius of the wheel, r_t is the radius

of the torodial cross section. Let $[m_{txi} \ m_{tyi} \ m_{tzi}]^T$, $[m_{nxi} \ m_{nyi} \ m_{nzi}]^T$ be the unit moment vectors due to \bar{T}_i and \bar{N}_i respectively .Now

$$A.C = D \quad (7)$$

Where $D = [F_x \ F_y \ F_z \ M_x \ M_y \ M_z]^T$,

$$C = [|\bar{T}_1| \ |\bar{N}_1| \ |\bar{T}_2| \ |\bar{N}_2| \ |\bar{T}_3| \ |\bar{N}_3| \ |\bar{T}_4| \ |\bar{N}_4|]^T$$

$$A = \begin{bmatrix} t_{x1} & n_{x1} & t_{x2} & n_{x2} & t_{x3} & n_{x3} & t_{x4} & n_{x4} \\ t_{y1} & n_{y1} & t_{y2} & n_{y2} & t_{y3} & n_{y3} & t_{y4} & n_{y4} \\ t_{z1} & n_{z1} & t_{z2} & n_{z2} & t_{z3} & n_{z3} & t_{z4} & n_{z4} \\ m_{tx1} & m_{nx1} & m_{tx2} & m_{nx2} & m_{tx3} & m_{nx3} & m_{tx4} & m_{nx4} \\ m_{ty1} & m_{ny1} & m_{ty2} & m_{ny2} & m_{ty3} & m_{ny3} & m_{ty4} & m_{ny4} \\ m_{tz1} & m_{nz1} & m_{tz2} & m_{nz2} & m_{tz3} & m_{nz3} & m_{tz4} & m_{nz4} \end{bmatrix},$$

$$\text{Let } \min(S), S = \sum_{i=1}^4 |\bar{T}_i| \quad (8)$$

$$|\bar{N}_i| \geq 0 \quad (9)$$

$$\Gamma_i^{\min} \leq (|\bar{T}_i|, r) \leq \Gamma_i^{\max} \quad (10)$$

Γ_i^{\max} , Γ_i^{\min} are the maximum and minimum torques that the motor can generate.

For the vehicle to move forward and remain in equilibrium we set $F_y \geq 0$,

and $|\bar{M}| = 0$. To depict the regions of infeasibility we solve (8) subject to (7),

(3), (9) and (10) to minimize motor torques. For the simplicity of analysis we

assume the front wheels are at contact angle γ_1 and rear wheels are at contact angle γ_2 which are varied from 0 to $\pi/2$ and ψ is varied from 0 to $(\gamma_1 - \gamma_2)$.

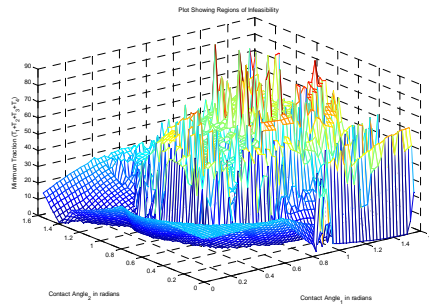


Fig.1.Plot of $\min(T_1+ T_2 +T_3+ T_4)$ Vs contact angles showing regions of infeasibility.

Fig 1 shows the plot of $\min(\sum_{i=1}^4 |\bar{T}_i|)$ as a function of the contact angles γ_1, γ_2 .

The discontinuities show the regions of infeasibility.

3. ANALYSIS OF LFA-V

For controlling the contact forces we develop an actively articulated suspension system and exploit its internal degree of mobility by an actuated prismatic joint through a linear force actuator mounted on the chassis. To develop our model we use a generic platform consisting of a chassis, prismatic joints and four toroidal wheels each pinned to an outer slide link of a prismatic joint, where as the inner slide link is fixed to the chassis. The forces that act at the i^{th} wheel of the vehicle are normal force \bar{N}_i , traction force \bar{T}_i and actuator force \bar{F}_i^A which is always perpendicular to the chassis. Now a similar analysis is done to frame the Quasi Static analysis for LFA-V.

The actuator force is given by $\bar{F}_i^A = \frac{|\bar{N}_i|}{\text{dot}(\hat{n}_i, \hat{f}_i^A)} \hat{f}_i^A$, $\hat{f}_i^A = R.[0 \ 0 \ 1]^T$.

Remark: Along the sliding direction the only force to be considered is \bar{F}_i^A . The property of a prismatic joint with linear force actuator is that only the components of \bar{T}_i and \bar{N}_i perpendicular to \bar{F}_i^A get transmitted to the chassis. To find the components of \bar{T}_i perpendicular to \bar{F}_i^A we find \bar{R}_{tai} which is resulted when \hat{f}_i^A is rotated by $\pi/2$ radians towards \hat{t}_i about $\bar{K}_{ti} = \text{cross}(\hat{t}_i, \hat{f}_i^A)$ [9]. Hence the component \bar{T}_i perpendicular to \bar{F}_i^A is given by $\bar{T}_{neti} = \text{dot}(\hat{t}_i, \bar{R}_{tai}) |\bar{T}_i| \bar{R}_{tai}$. Similarly component of \bar{N}_i perpendicular to \bar{F}_i^A is given by $\bar{N}_{neti} = \text{dot}(\hat{t}_i, \bar{R}_{nai}) |\bar{N}_i| \bar{R}_{nai}$. Let

$$\hat{t}_{neti} = \frac{\bar{T}_{neti}}{|\bar{T}_{neti}|} = [t_{netxi} \ t_{netyi} \ t_{netzi}]^T \text{ and } \hat{n}_{neti} = \frac{\bar{N}_{neti}}{|\bar{N}_{neti}|} = [n_{netxi} \ n_{netyi} \ n_{netzi}]^T.$$

Hence the net force and net moment is given by

$$\bar{F} = \sum_{i=1}^4 (\bar{F}_i^A + \bar{T}_{neti} + \bar{N}_{neti}), \quad \bar{M} = \sum_{i=1}^4 [(\bar{r}_{fai} \times \bar{F}_i^A) + \bar{r}_{mi} \times (\bar{T}_{neti} + \bar{N}_{neti})]$$

Now a similar quasi static analysis is done for LFA-V. From the Fig.2 it is easy to see that all infeasibility regions are eliminated. To attain a desired posture, the vehicle parameters that need to be controlled while traversing a terrain are the

height h of the chassis, velocity V in Y direction, ψ , ro and yo . To achieve desired velocity V_d for the vehicle we command a suitable value of $F_y = \bar{k}_p e_v + \bar{k}_d \dot{e}_v$, $e_v = V_d - V$. Similarly we have $F_z = k_p e_h + k_v \dot{e}_h + Mg$, $M_x = \bar{k}_p e_\psi + \bar{k}_v \dot{e}_\psi$, $M_y = \hat{k}_p e_{ro} + \hat{k}_v \dot{e}_{ro}$, $M_z = \tilde{k}_p e_{yo} + \tilde{k}_v \dot{e}_{yo}$. $e_{ro} = ro_d - ro$, $e_h = h_d - h$, $e_\psi = \psi_d - \psi$, $e_{yo} = yo_d - yo$. Where $e_v, e_h, e_\psi, e_{ro}, e_{yo}$ and e_{yo} are the differences between the desired values and the instantaneous values of the parameters. $\bar{k}_p, \bar{k}_v, \hat{k}_p, \hat{k}_v, \tilde{k}_p, \tilde{k}_v$ and $k_p, k_v, \hat{k}_p, \hat{k}_v, \tilde{k}_p, \tilde{k}_v$ are the proportional and derivative gains respectively. Control equations are developed to overcome the differences in the dynamics of the system which arise due to the assumption of negligible wheel and link mass when compared with the chassis.

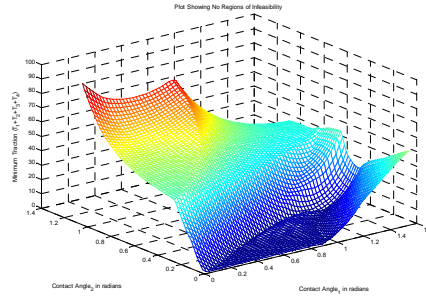


Fig. 2 Plot of $\min(T_1 + T_2 + T_3 + T_4)$ vs Contact angles showing regions of infeasibility are eliminated

4. RESULTS AND DISCUSSION

Fig.3 and Fig.4 shows simulations for the terrains 1,2 which are modeled such that every point on the surface has a finite and unique gradient in any three orthogonal directions. These simulations were performed using MATLAB, Simulink and MSC Visual Nastran. We obtain $\bar{N}_i, l_i, V, \psi, ro$ and yo from MSC Visual Nastran. The controllers were applied to maintain $V_d = 0.5\text{m/s}$, $\psi_d = ro_d = yo_d = 0$ and $h_d = 0.42\text{m}$ by assuming $M = 9.31\text{ Kg}$, $a = 0.3\text{m}$, $w = 0.2\text{m}$, $r_t = 0.0125\text{ m}$ and $r = 0.05\text{m}$. Fig.5 and Fig.6 show the plots of Euler angles for terrain 1, 2. It is easy to see that the deviations of these angles are well within acceptable limits.

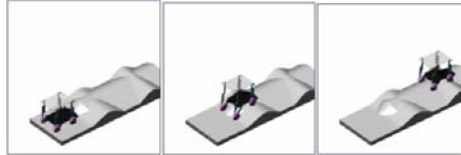


Fig. 3 LFA-V negotiating terrain 1



Fig. 4 LFA-V negotiating terrain 2

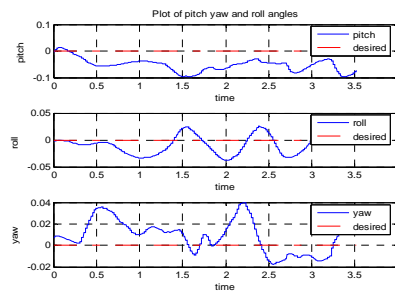


Fig 5 .Plot of Pitch, Roll and Yaw - terrain-1

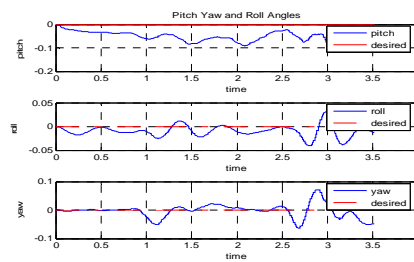


Fig 6 .Plot of Pitch, Roll and Yaw - terrain-2

5. CONCLUSIONS

In our work we present LFA-V which negotiates uneven terrain by modifying the contact forces while maintaining a desired posture. Such an approach for rough terrain mobility does not seem to have reported in the literature. The paper

presents a quasi static analysis for the system and also a motivation for using LFA-V over and above a passive suspension system by depicting enhanced feasibility regions. The efficacy of this method is confirmed by the plots of Euler angles which are well within the acceptable limits ensuring desired posture. From the noise analysis the system is stable for reasonable values of sensor noise at the points where contact forces are measured.

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