Active Localization of Multiple Mobile Robots By Moving to Best Frontiers

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***Abstract* —We envisage a multi robotic scenario where several robots are in ambiguity about their states and require help of other robots to overcome their ambiguity. Ambiguity here is used in the sense of more than one hypothesis of a robot's state. In such a scenario the method presented here moves the robots to locations where probability of eliminating several ambiguous states among multiple robots is a maximum. For this purpose the best frontiers are identified and robots dispatched to those frontiers. The best frontiers are those that have the highest probability of realizing a unique hypothesis if the robots were to arrive there. The method presented has been tested in both simulation and real-time on robots and its efficacy verified. Extensive comparative analysis portrays the advantage of the current method over others that do not perform active localization in a multi-robotic sense.**

I. INTRODUCTION

We consider a scenario of multiple robots navigating in a known map end up with multiple hypotheses of their states when attempting to globally localize in the given map. This is a pathological problem in global localization and is well documented [1,2]. This problem is attacked for the case of a single robot by moving to locations where they can best localize in [1,4] and the terminology active localization was introduced in [1]. This paper tackles the active localization problem for multiple robots. Specifically given a set of robots, each of them having more than one hypothesis of their states, we propose a method that moves robots to places such that a maximum number of such robots singularize to a unique state. Intuitively robots would want to move to places where a large number of them can detect one another. However mere detection of large numbers does not suffice unless the detection happened from locations where the measurements were unique and cannot be replicated elsewhere between robots. Since robots' initial states are multi modal in nature it is not possible to enumerate places where unique measurements between robots occur. Hence a *probabilistic framework* is developed that finds the probability of reaching to places where unique measurements leading to unique hypotheses is maximum for a set of robots. These positions are frontiers in the sense used in [3]. The probabilistic framework also accounts for other constraints such as the detecting range of the sensors as well as presence of obstacles. These are detailed in section 3. For example Figure1a tries to globally localize the robot in a given map. The state of the robot is a uniform distribution portrayed by the dark cells. Each such cell is a possible pose of the robot. In Figure 1b after the robot acquires a scan the possible pose of the robot is multi modal in nature shown once again by the clusters of dark cells.

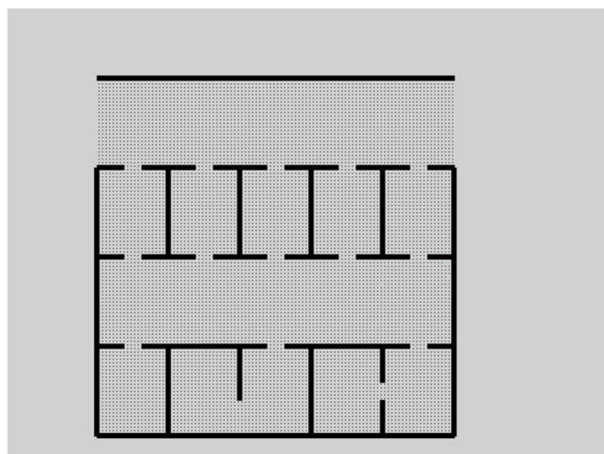


Figure1a tries to globally localize the robot in a given map. The state of the robot is a uniform distribution portrayed by the dark cells.

Extensive comparative analysis portrays the advantage of the current method over others that do not perform active localization in a multi-robotic sense. The method presented here would find utility in several multi-robotic applications where localization issues dominate. It is useful when robots brought alive need to estimate their initial state. It is useful when several robots tend to get confused about their states due to large localization errors and need to globally localize once in a while in the map provided or built so far. It is once again useful if one or more robots in a multi-robotic endeavor could get kidnapped by malicious agents. While the context mentioned here is resolving multiple hypotheses among multiple robots this effort can also be seen in the larger context of cooperative localization where robots also serve the function of mobile landmarks and come together to estimate their state whenever required. The authors argue that the novelty of this work lies in that it is the first such work to be reported where multiple hypothesis states in more than one robot is resolved in an active localization sense.

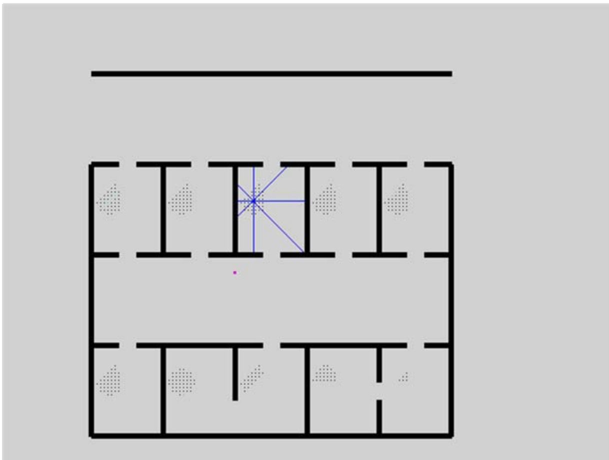


Figure 1b after the robot acquires a scan the possible pose of the robot is multi modal in nature shown once again by the clusters of dark cells.

II. RELATED WORK

In general work on active localization has tended to be limited when compared with passive localization. The pioneering work in this area has been from [1] and [4]. In [1] a method of active localization based on maximum information gain was presented. Dudek and others [4] presented a method for minimum distance traversal for localization that works in polygonal environments without holes that they show to be NP-Hard. A randomized version of the same method was presented in [5]. In [6] an approach for guiding the robot to a target location is proposed when its current position is not known accurately while [11] presents a method of localizing a single robot in multiple states based on exploration and selecting best directions of motion.

As far as authors knowledge goes there has been no reported work dealing active localization of several robots. In the context of multi robotic localization Fox and others extended their earlier global localization methods to a multi-robotic setting in [7]. A cooperative localization approach was also presented based on Monte-Carlo methods in [8]. In [9] a method is presented in which team members carefully coordinate their activities in order to reduce cumulative odometric errors. However these methods did not consider active localization in a rigorous sense of guiding robots to positions.

III. METHODOLOGY

Consider a workspace W populated by robots, N_R in number, each of them having multiple hypotheses of their states. The problem is to move these N_R robots to frontier locations such that each of the robots localizes to a unique state with minimal number of frontiers visited.

The problem is approached in the following fashion.

i. Firstly we find the probability of robots reaching or occupying frontiers (occupancy probability) from where unique measurements between robots are possible.
ii. Secondly the occupancy probability is modified incorporating visibility constraints to compute the actual probability of obtaining unique hypothesis for a robot if it reached those frontiers.
iii. Since several combinations of robots occupying frontiers that gives rise to unique hypothesis is possible that combination is chosen that maximizes an objective function.

A. Computing Occupancy Probabilities

We introduce the notion of distinct and virtual frontiers here. Consider the map of figure 2 in the form of two almost identical rooms with two openings one in the top and one at the bottom. A globally localizing robot in this map finds two hypotheses locations in H1, H2 (figure 2). At each of these hypotheses position there are two frontiers for the robot, one in the top and other at the bottom, labeled ft1 and fb1 for H1. These frontiers at each hypothesis location are computed as in [3] and are called distinct frontiers. If a robot has n_h possible hypotheses then each distinct frontier has n_h such copies called virtual frontiers. In figure 2 each distinct frontier at top and bottom has two virtual frontiers ft1, ft2 and fb1, fb2. Henceforth the term frontier refers to a virtual frontier unless explicitly mentioned as distinct.

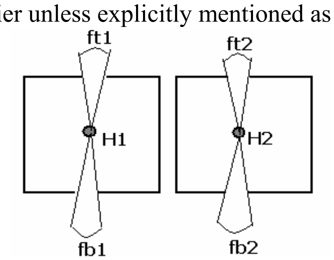


Figure2: A robot with two hypotheses states H1 and H2. The distinct frontiers for each state are two one on top and other at bottom denoted as ft1 and fb1 for state H1 at the boundary of the visibility polygon. There are two virtual frontiers (ft1, ft2) for every distinct frontier due to two hypotheses states.

Imagine a map where there are n such similar rooms as in figure 2. Then a globally localizing robot would have n hypotheses. If it decides to occupy the distinct frontier in the top (we assume the robot's orientation is known through a compass and only its coordinates are unknown) it would end up reaching one of those n virtual copies in $1/n$ ways. More formally if there is a set of robots R with cardinality n_R capable of occupying a set of frontiers F, with cardinality n_f the probability a particular k of those, say $f_1, f_2, \ldots, f_k$ of them gets occupied is given by

$$P_o(n_f, n_R, k) = \begin{cases} \dfrac{Pm(n_R, k)Pm(n_f - k, n_R - k)}{Pm(n_f, n_R)} & ; n_R \geq k \\ 0 & ; n_R < k \end{cases} \ldots\ldots(1)$$

Here the term $P_o(n_f, n_R, k)$ denotes the probability of occupancy of a particular k of the n_f frontiers by n_R robots. The first term in the parenthesis denotes the number of frontiers, the second the number of robots and the third number of frontiers to be occupied. The notation $Pm(x, y)$ denotes the permutation of x things taken y at a time. Among the various possible arrangements of robots some of them give rise to unique measurements between robots. Specifically we look for a pair of frontiers that give rise to a unique measurement. A measurement between two frontiers is denoted by (d, θ) and measures the distance and angle between the two. A *unique measurement* is one that is unique in terms of either d or θ between a pair among all pairs of frontiers considered. For example in a symmetric five room suite in figure 3 the initial global localization algorithm gives rise to five possible hypotheses for each of the robots, say two, in those rooms. The only pair of unique measurement between frontiers is when the end frontiers at the top get occupied (figure 3) since the frontier locations are equally spaced. The probability of occupancy of the two end frontiers from the two robots is the computation of $P_o(5,2,2)$. For measurements between any other pair of frontiers get replicated elsewhere. For example measurement between first and second frontiers is the same as between 2 and 3, 3 and 4 & 4 and 5 since they are spaced equally. *Given a set of robots capable of occupying certain locations it suffices to look for locations that give rise to a unique measurement for obtaining a unique hypothesis. Once the robots at those locations are uniquely localized they can localize other robots in that set either directly (if they are visible) or through a chain of robots acting as intermediaries as explained below.*

B. Unique Hypothesis Probabilities and Visibility Constraints

For a set of robots occupying a set of frontiers unique measurements can be obtained for more than a pair of frontier locations. For example in figure 3 for the suite of 5 rooms let the distance between frontiers 2 and 3 be different while all the distances between adjacent frontiers namely 1,2; 3,4; 4,5 be same. Then the following pairs of frontiers gives rise to unique measurements between them namely 1,3; 1,4; 1,5; 2,3; 2,4; 2,5 & 3,5 that are the objects of the set *UP* that contain all the pairs of frontiers with unique measurements. The probability of obtaining a unique measurement between frontiers 1,3 is merely the probability of occupancy of those two frontiers as computed in previous section. The probability of obtaining a unique hypothesis for any robot in the set is the union over all those probabilities of occupancies that give rise to unique measurements.

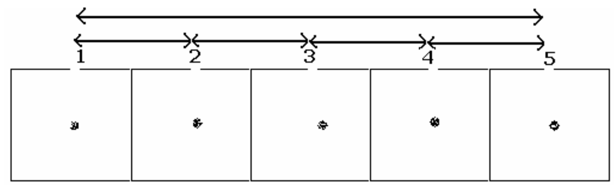


Figure 3: A suite of 5 identical rooms with an opening at the top. Occupancy of the extreme frontiers by robots gives rise to a unique measurement that does not get replicated elsewhere. This is shown by the longest arrow at the top. Occupancy of adjacent frontiers however gets replicated for four other placements as shown by the shorter arrows

More formally given a set of robots R and a set of frontiers F denote the probability of obtaining a unique hypothesis for any robot in this set by $P_{UH}(FR)$ given that there are no frontiers that are outside of this set that will be occupied by a robot in *R*. *F, R* denotes the frontier and robot sets for which the computation is done. Also denote by $P_{UM}(p)$ the probability of obtaining a unique measurement due to a pair of frontiers p belonging to the set *UP*. $P_{UM}(p)$ is nothing but the probability of occupancy of that pair of frontiers as computed in the previous section. Then

$$P_{UH}(FR) = \bigcup_{p \in UP} P_{UM}(p) \ldots\ldots(2).$$

While computing the union the intersections are dealt till order five since terms of order greater than five are insignificant upon extensive experimentation.

C. Incorporating Visibility Constraints

While computing the probability of obtaining a unique measurement the implicit assumption has been the robots occupying the frontier pair are visible to one another (there are no obstacles between) and are within sensing range of each other. When two frontiers are within sensing range the presence of a robot in between the two occupied frontiers does not in any way change any of the above probability computations. When robots detect each other through a unique measurement they get localized to a unimodal distribution of their states. The presence of other robots in between does not prevent this as explained through figure 4 that shows a set of five frontiers equally spaced and abstracted as circles. The only unique measurement is when the extreme frontiers are occupied. Consider a third robot occupying frontier three. After belief update according to the Markov framework of [1] between robots r1 and r3, r1's possible positions are at frontiers 1 and 3, while that of r3 are at 3 and 5. After belief update between r2 and r3, r2 gets localized at location of frontier 5 and r3 at 3. Another belief update between r2 and r1 localizes r1 at 1. This is equivalent to the situation when r1 and r2 detected each other directly. How multiple robots update their beliefs between each other is not discussed here and we cite [10] for this purpose. In short visibility constraints need to be looked into only when robots at two frontiers are not within sensing range of each other or presence of an obstacle in between. We discuss both these cases through simple examples.

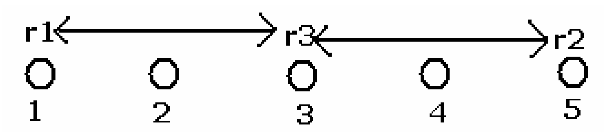


Figure 4: Five equally spaced frontiers numbered 1 … 5 and abstracted as circles. If robots r1 and r2 occupy the extremes and detect each other through a unique measurement they localize to unimodal states. The presence of a third robot r3 at 3 does not in any way affect the result despite r1 and r2 unable to detect each other directlv

Visibility constraints due to obstacle presence

Consider figure 5 which shows 6 frontiers abstracted as circles and numerically labeled. We want to find the probability of obtaining a unique measurement between frontiers 2 and 6 due to occupancy of those frontiers by robots r1 and r2. The presence of an obstacle between frontiers 2 and 5 prevents a unique measurement. However a unique measurement between the two that leads to a unimodal state for both is possible if the remaining robots r3 and r4 occupy any two of the three frontiers 3, 4 and 5. Thus the probability of obtaining a unique measurement between 2 and 5 is the union of the following occupancy probabilities: (i) The probability of 2,3,4 and 6 being occupied (ii) The probability of 2,3,5 and 6 being occupied and (iii) The probability of 2,4,5 and 6 being occupied. Each of the above probabilities is of the form $P_o(6,4,4)$ and can be computed as before. Thus the effect of visibility constraints invokes placement of other robots in various positions such that the robots at the considered frontier pair are still able to localize to a unimodal state by propagating beliefs through others. More formally each term in (2) that considers the probability of obtaining a unique measurement due to a pair of frontiers is now further decomposed to a union of terms. Each term in this union denotes the placement of other robots at frontiers apart from the considered pair such that the considered pair of robots localize.

Visibility constraints due to farness of frontiers

The effect of a frontier pair being far away for robots to detect is dealt in the same vein as before. Consider figure 4 again with the added constraint that the maximum detection range for any robot is three times the distance between the two adjacent of the equally spaced frontiers. Albeit r1 and r2 unable to detect one another if r3 occupies frontier 3 then r1 and r2 would still be able to localize through a unique measurement with the help of r3. Thus the probability of obtaining a unique measurement between frontiers 1 and 5 is identical to the probability of frontiers 1, 3 and 5 being occupied when there are 3 robots. This computation is of the form $P_o(5,3,3)$

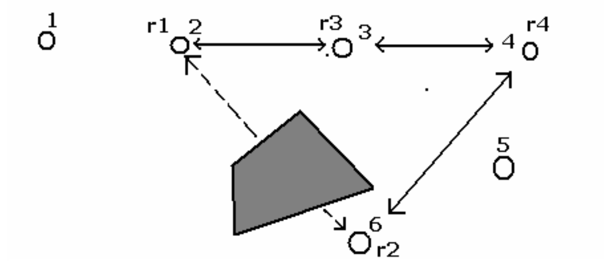


Figure 5: Though robots r1 and r2 at 2 and 6 cannot detect one another due to the presence of the obstacle, the occupancy of r3 and r4 at any of the locations 3, 4 and 5 would have the same effect of r1 and r2 detecting each other directly. If r1 and r2 would have localized through a unique measurement in the absence of the obstacle, they still could if r3 and r4 occupy of two of the frontiers 3, 4 and 5.

D. Choosing the Best Combination

Since the number of combinations of robots with frontiers is *exponential* in the number of robots, the original set of robots R is now partitioned into *n* mutually exclusive subsets $R_1, R_2, \ldots, R_n$ where the cardinality of set R_i is n_{Ri} and $\sum_i n_{Ri} = N_R$. The partition is done in a manner such that all robots in the partition share the same set of distinct and hence virtual frontiers. By sharing we mean for every virtual frontier f_{ri} for robot $r_i \in R_i$ there exists exactly one virtual frontier f_{rj} for every other robot $r_j \in R_i, r_j \neq r_i$ that is close to f_{ri} in a Euclidean sense, their distance less than a threshold. If such frontier is not found among other robots then r_i is the only member of R_i. We call each such partition of robots as a base pair. The number of base pairs equals the number of mutually exclusive subsets. We denote base pair *i* as fr_i, *f* indicates that all robots in the base pair share the same frontiers and *r* indicates the robots themselves. The partitioning reduces the number of combinations to be considered while moving to best frontier locations. It then suffices to take a decision of where to move next for just one robot in the base pair, which is then replicated for all others in that base pair.

Each such base pair has a list of distinct frontiers as possible locations to visit and localize. We denote by set $DF_i = \{fr_i^1, fr_i^2, \ldots, fr_i^d\}$ the *d* distinct frontiers for the base pair fr_i. The superscript refers to the index of the distinct frontier for the base pair fr_i. The list of all possible combinations is enumerated through the operation $DF_i \times DF_{i+1}; i = 1 \cdots n-1$. For each combination we compute the probability of obtaining a unique hypothesis based on the frontiers and robots that constitute the base pairs of that combination. The combination that has the highest probability of obtaining a unique hypothesis is considered and the robots are dispatched to frontiers accordingly. The computations are based on equation (2) slightly modified to handle combinations of base pairs than combination of robots. We don't report for the sake of brevity.

IV. SIMULATION RESULTS

Figure 6a shows five suites labeled S1, S2… S5. Each suite is composed of three or more rooms. The initial positions of the robots are more or less at the center of the rooms. There is one robot r1 in S1, r2 and r3 in S2, r4 and r5 in S3, r6, r7 and r8 in S4 and r9, r10 and r11 in S5. All the robots shown have multiple hypotheses of their states initially. The hypotheses states of robots are shown as darkened cells (the clusters of dark points). Robot r1 has 2 hypotheses, r2, r3, r4, r5 have 4 hypotheses and r9, r10, and r11 have 5 while the remaining robots have 3 hypotheses. All of them possess two distinct frontiers one at the top and one at the bottom for robots in S1, S2, S3 and S5 while the robots in S4 have distinct frontiers on left and right at each of their hypothesis position. Each of the two distinct frontiers has as many copies of virtual frontiers corresponding to the number hypotheses states.

There are five base pairs $fr_1, fr_2, \cdots fr_5$ with the superscript *t(l)* and *b(r)* indicating the top(left) and bottom(right) distinct frontiers in each base pair. Robots r1 constitute the base pair fr_1 , r2 and r3 form fr_2 , r4 and r5 with fr_3 , r6, r7 and r8 in fr_4 and r9, r10 and r11 in fr_5 .

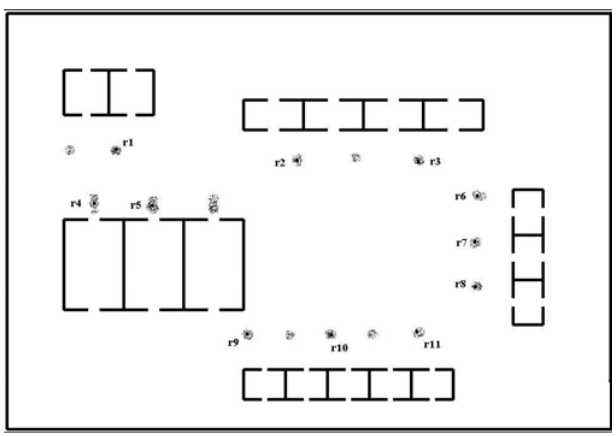


Figure 6(b) shows the scenario after the robots moving to the corresponding frontiers selected. Once they reach those frontiers motion and sensor updates take place which could localize only robot r9. For other robots the numbers of hypotheses have reduced.

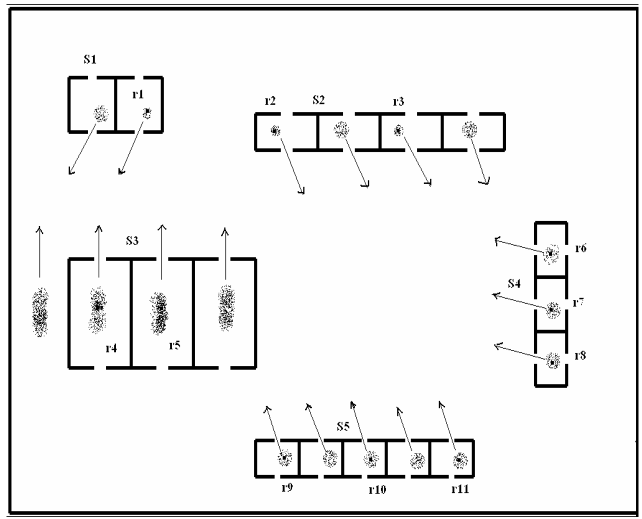


Figure 6(a) shows 5 suites of rooms labeled S1, S2, S3, S4 and S5 with robots r1, r2 - r3, r4 - r5, r6 - r8, r9 - r11 respectively. It also shows the hypotheses for each robot. Arrows show the movement of robots to corresponding frontiers.

Figure 7 tabulates *some* of the combinations for the figures in 6. The corresponding number of unique measurements between frontier pairs and the probabilities of unique hypotheses for each combination is also shown. The combination $\{fr_1^b, fr_2^b, fr_3^t, fr_4^l, fr_5^t\}$ has the highest probability. The arrows show the direction in which each robot moves as they reach the frontiers corresponding to the best combination. Upon reaching their respective frontiers and performing a sensor update only r9 got localized because of the nature of the obstacle configuration seen by it locally while r2… r5 had their number of hypotheses states reduced from 4 to 3. The situation corresponding to

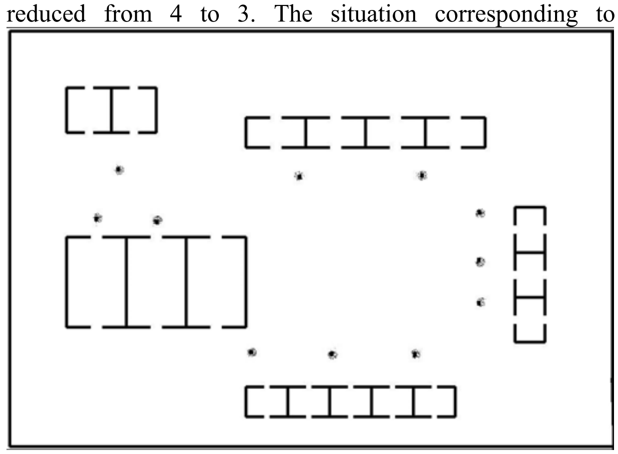


Figure 6(c) shows the scenario after the robot-robot detection and updates. All the robots in the map got localized i.e. they have unique hypothesis now.

this is shown in figure 6b. However when robots detected one another and performed updates all of them localized to a

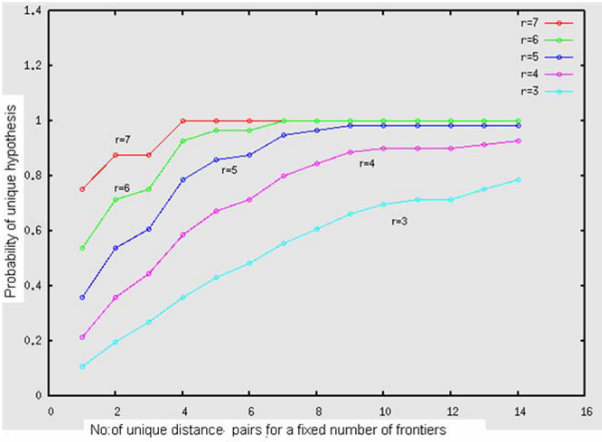


Figure 8: Graph showing probability of obtaining a unique hypothesis versus number of unique measurements for a fixed number of frontiers for different numbers of robots

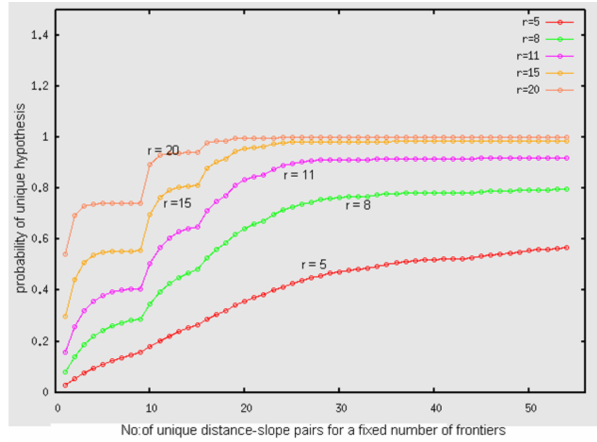


Figure 9: Graph showing probability of obtaining a unique hypothesis versus number of unique measurements for a fixed number of frontiers for different numbers of robots

unique hypotheses state preventing any further entailment to explore and localize (figure 6c). This shows that robots reaching specific locations and performing updates based on their detections helps a long way in quick localization.

Combinations	No: of Unique measurements	Probability
$\{f_1^t, f_2^t, f_3^t, f_4^l, f_5^t\}$	39	0.0661150
$\{f_1^t, f_2^t, f_3^t, f_4^r, f_5^t\}$	41	0.0301914
$\{f_1^t, f_2^t, f_3^b, f_4^l, f_5^t\}$	47	0.09733595
$\{f_1^t, f_2^t, f_3^b, f_4^r, f_5^t\}$	46	0.03769381
$\{f_1^t, f_2^b, f_3^t, f_4^l, f_5^t\}$	85	0.51914762
$\{f_1^t, f_2^b, f_3^t, f_4^l, f_5^b\}$	33	0.02481619
$\{f_1^b, f_2^t, f_3^t, f_4^l, f_5^b\}$	52	0.1129033
$\{f_1^b, f_2^b, f_3^t, f_4^l, f_5^t\}$	125	0.97764285
$\{f_1^b, f_2^b, f_3^t, f_4^l, f_5^b\}$	61	0.0853095
$\{f_1^b, f_2^b, f_3^t, f_4^r, f_5^t\}$	129	0.5692
$\{f_1^b, f_2^b, f_3^b, f_4^l, f_5^t\}$	110	0.93391666
$\{f_1^b, f_2^b, f_3^b, f_4^r, f_5^t\}$	112	0.4402119
$\{f_1^b, f_2^b, f_3^b, f_4^r, f_5^b\}$	54	0.0634376

Figure 7: Tabulation of the some of the base-pair combinations for the map shown in figures 6.

A. Generic Trends

Figure 8 shows the graph of probability of obtaining a unique hypothesis versus number of unique measurement pairs for a fixed number of frontiers (here 12) for various number of robots occupying those frontiers. The probability increases as the number of unique measurement pairs increase for a given number of robots. Also across the robots the probability increases as more number of robots occupies the same space. This graph sums up the essence of this effort; it gives a bird's eye view of where the robots should move to maximize probabilities of obtaining a unique hypothesis. Figures 9 also capture the same trend for different maps.

B. Comparative Analysis

We compare our method with three different methods. In the first method robots move randomly and do not detect each other, in the second robots move according to the method given in [11] that selects best frontiers to move from a single robotic perspective but neglects robot-robot detections. In third robot-robot detections are considered but chose frontiers randomly. The graphs of figure 10 and 11 portray the comparisons. In both the graphs method 1 is the method proposed in this paper. Method 2 is the one which chooses frontiers randomly to move to with robot-robot detection and updates. Method 3 is the method given in [11] that selects best frontier to move from a single robotic perspective but neglects the robot-robot detections. In Method 4 robots move randomly and do not detect each other.

Figure 10 plots the number of explorations (frontiers to be reached) on the abscissa and the number of robots that haven't localized to a unique state on the ordinate. Graphs are the results of simulations performed over several maps and averaged over several runs, each map having the same number of robots, 11, at start initially. Clearly our method has all the robots localized with least number of explorations or frontiers to visit while the methods that do not incorporate robot-robot detection at all take a long time to converge or sometimes do not converge at all depending on the given map. The method that takes into account robot-

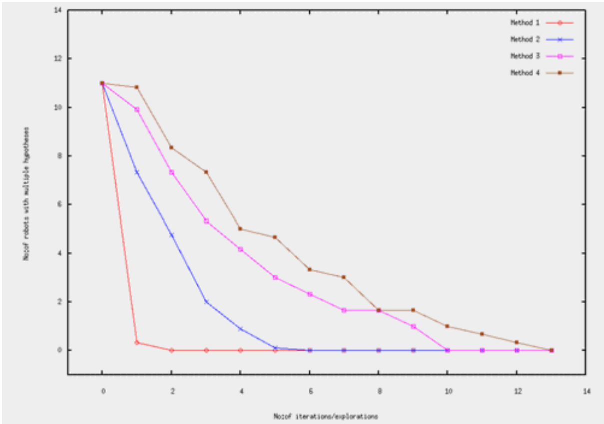


Figure 10: Graph showing number of iterations versus number of robots with multiple hypotheses (robots remaining to be localized) for a fixed number of robots averaged over different maps.

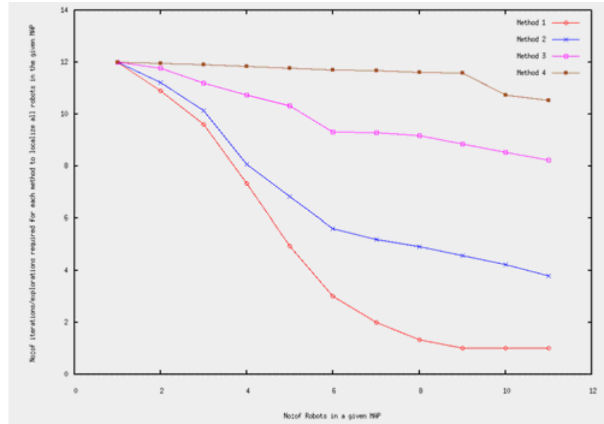


Figure 11: Graph showing number of robots in a given map versus number of iterations for each method to localize all the robots in that map (averaged over a large no: of maps).

robot detection but chooses frontiers randomly also takes a much longer time, in comparison with our method, to localize all robots to a unique state.

Figure 11 captures the trend of increasing number of robots visiting the same area across methods. It plots number of robots on the abscissa and number of explorations on the ordinate. As the number of robots increase the number of explorations decrease for methods that make use of robot-robot detection updates.

Evidently the current method localizes faster than the method 2 (which chooses randomly frontiers to move to). Since methods 3 and 4 do not incorporate robot-robot detection and update, increasing the number of robots does not have a prominent effect. Figure 11 reinforces the trends of figures 8 and 9.

On extensive simulations it is observed that the localization of robots depends on the factors like sensor range, visibility range, number of robots present in the environment and also on the initial states of the given robots in the given map. Depending on the sensor range and the visibility range the current method takes 1-2 explorations (iterations) to localize all the robots. The dependency of each method on these factors grows stronger as we go from method 1 to method 4. In other words the method suggested here proved to be most robust to the initial states of the robots in a given map. It is also observed that in the event the current method can not localize all the robots for a given exploration it ends up with a lesser number of hypotheses for the robots which are not localized than the other methods. This is expected because the theoretical frame models the probabilistic nature of the initial states of the robots (all robots have multiple hypotheses initially) and gives direction based on where the probability of arriving at unique hypothesis is maximum. As expected the results show that increase in number of robots in the same environment hardly has any effect on the methods which do not use robot-robot detection and updates.

V. IMPLEMENTATION

The method was verified on a pack of Amigobots equipped with 8 sonar transducers. External hardware in the form of IR transceiver circuit was interfaced to the serial buffer of the Amigobot's controller board to facilitate easy detection of one robot by other. Each transceiver transmitted a unique pulse code corresponding to a particular robot. It also received similar codes corresponding to other robots. The hardware apart from detecting a robot was capable of measuring the bearing between the detected and detector robot. Sonar sensors are then fired along the detected direction to measure distance between the two. Figure 12 shows the external hardware interfaced to the controller on the Amigo.

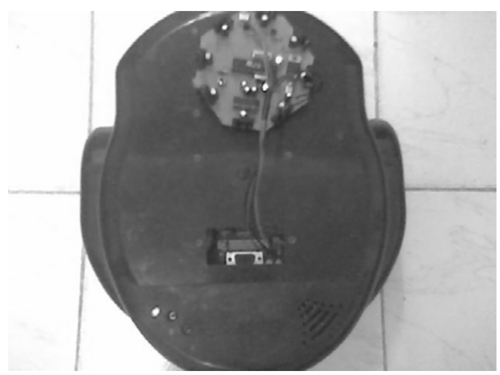
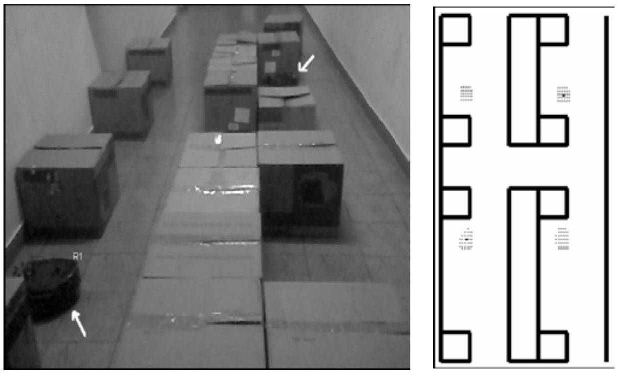
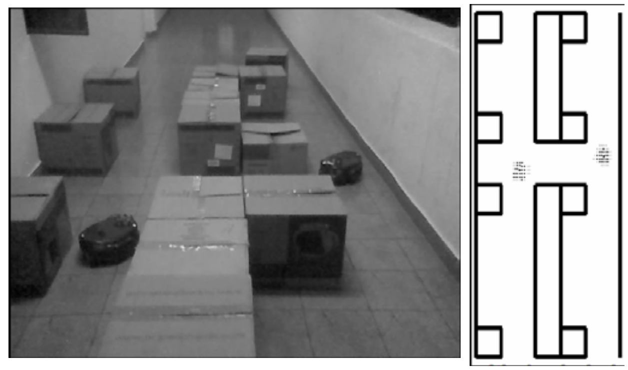


Figure 11: The external hardware interfaced to the controller on the Amigo.

Figure 13a shows a corridor studded with cardboards somewhat akin to situations in warehouse with two Amigos. Figure 13b shows the hypotheses returned by the global localization procedure resulting in two distinct frontiers (one



Figures 13a-b: 13a: Two robots (indicated by white arrows) in an environment studded with cartons. 13b: The hypotheses states corresponding to 13a.



Figures 13c-d: 13c: The robots detect one another through IR transceivers upon nearing the frontiers. 13d: The hypotheses states corresponding to 13c with each robot having now singularized to a unique state

at the top and one at bottom) for each robot. Each frontier has two copies of virtual frontiers corresponding to the two hypotheses states for a robot. The number of base pairs is thus two and the combination $\left\{fr_1^b, fr_2^t\right\}$ yielded the maximum probability. The robots are dispatched to the corresponding frontiers and upon reaching detect one another in figure 13c. The unique hypothesis states corresponding to figure 13c is shown in 13d for the two robots. It is to be noted however that the robots localized before detecting one another due to the nature of the asymmetries present in the environment.

VI. CONCLUSION

This paper has presented a novel method of shepherding robots to locations for rapid elimination of multiple hypotheses among them. A set of robots R_i with cardinality n_{Ri} and sharing the same frontier set F_i is represented by the base pair fr_i in the paper. A combination of several such base pairs is considered and the combination with the highest probability of localizing to a unique state chosen and the robots shepherded to those locations to eliminate maximum number of conflicting hypotheses. The strategy has been tested both in simulation and real robots and its efficacy verified. Comparative analysis clearly portrays the advantage of the current method vis-à-vis others that do not incorporate the current strategy. This method finds utility in several multi-robotic scenarios, where robots are not clear about their state, have multiple hypotheses and require the assistance of other robots to resolve conflicts as well as to refine their states to precise coordinates.

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