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Sensor Based Localization for Mobile Robots by Exploration and Selection of Best Direction

Rakesh Goyal
Robotics Research Center
IIIT,Hyderbad,India 500032
rakesh_goyal@students.iiit.ac.in

K. Madhava Krishna
Robotics Research Center
IIIT,Hyderbad,India 500032
mkrishna@iiit.ac.in

Shivudu Bhuvanagiri
Robotics Research Center
IIIT, Hyderabad, India 500032
shivudu@students.iiit.ac.in

Abstract— **We present a strategy for resolving multiple hypotheses of a robot's state during global localization. The strategy operates in two stages. In the first stage a unique direction of the motion is sought that resolves or eliminates maximum number of hypotheses. In the second stage, among the frontier areas arising from the multiple hypotheses states, that frontier is chosen which resolves the maximum number of the hypotheses. The two stages are alternated till a unique hypothesis emerges. Simulation and experimental results verify the efficacy of this method. A comparison with other methods based on entropy minimization, and minimum distance travel portrays the advantage of the current methodology. A convergence proof for the algorithm is also presented.**

Index Terms— **Mobile Robotics, localization, frontier exploration.**

I. INTRODUCTION

This paper attacks the problem of how to navigate a mobile robot to get rid of multiple conflicting hypotheses, within a global localization framework. Global localization is the problem of estimating the state (pose) of the robot in an *apriori* known map without an initial estimate of its state. It is inevitable that a global state estimation procedure will lead to several hypotheses of the robot state in a symmetric environment. Such a situation occurs commonly in indoor navigation such as in corridors and office rooms. In this context algorithms that move the robot to places where it can uniquely localize itself gain prominence. In other areas of AI(machine learning and heuristic search) the role of active control during learning and problem solving is often encountered [4], [5].

This paper presents a two-stage strategy for navigating the robot to quickly come up with a single hypothesis of its state. In the first stage a direction of motion is chosen along which a cost function is maximized. The cost function is framed in such a manner that the direction for which it is maximized is that direction along which either (i) the robot travels minimum distance to eliminate hypotheses, or (ii) maximum number of the hypotheses get eliminated, or (iii)a combination of (i) and (ii) by suitable setting of parameters. In the second stage, among the frontier areas arising from the multiple hypotheses states, that frontier is chosen which resolves the maximum number of the hypotheses. The two stages are alternated till a unique hypothesis emerges.

This approach is different and contrasts with other well cited approaches [1]–[3] of active localization in the following ways. The method of active localization presented in [1] is based on the principle of maximum information gain. The authors consider that for every potential relative movement, the maximum expected information gain is traded off by the expected path cost. However due to the problem being exponential in the number of actions or movements (since every action of the robot looks at only the 8 neighbouring grids) the number of actions is limited to one, or a very small number. Since the robot considers only 8 neighbouring grids in advance at any instant the robot traversal tends to become long often visiting cells that were previously visited. Also the entropy calculation required for every grid is intensive. The current approach is complete in that a path that can localize the robot is formed if it exists and the computation at each time step is much less as shown by the time comparisons divulged in section 4.

The method presented by Dudek et.al. [2] offers a complete solution for localizing a robot with minimum distance. Given a world map in the shape of a simple polygon P and the mobile robot's visibility polygon V, seen by the robot from its current location, their algorithm first finds a set of points in P whose visibility polygon is congruent under translation to V. This is the set of multiple hypothesis locations, denoted by H. The algorithm then computes a minimum distance strategy that is greedy and based on the distance to the new probe position and the information to be gained at the new location. Central to the algorithm is the computation of an overlay arrangement and a visibility cell decomposition of the arrangement. These computations have strong dependencies that the environment is a polygon made of vertices and edges with no holes inside. Hence, when the environment is a polygon with holes (such as a room with a single table in it) a variant of this method needs to be used that the authors have not come across in literature.The room is the polygonal environment. The tables are the obstacles that represent the holes in the polygon. In other words the projection of tables on the reference plane creates patches.

This method works with both these cases, since it relies only on raw range reading and the ability to compute the frontier in a generic environment.

II. Related Work

In general, work on active localization has tended to be limited when compared with passive localization. The pioneering approaches include [1]–[3], which has been discussed in detail in previous section. In [2], Dudek et. al. show that minimum distance localization is NP-Hard. In [6], a method is proposed for speeding up the implementation provided by [2]. An active localization method that models the environment as a bounded geometric tree is formulated in [7]. In [8] an approach for guiding the robot to a target location is proposed when its current position is not known accurately.

There have been numerous approaches to passive localization and many of them are archived in the scholarly book by Borenstien [9]. Passive localization methods can be broadly classified into global and local localization methods. Probabilistic localization [1], [10] approaches have been the flavor of the day in global localization. Local localization approaches have generally adopted Kalman filter [11], or estimate the state of the robot through error minimization schemes such as the least-squares [12] or scan matching [13] There is also a vast body of literature on SLAM that is beyond the scope of this paper and not refered due to brevity of space.

III. Methodology

Given: A robot equipped with range sensor whose model is known and a map which is symmetric.
Objective: To find out the pose of the robot by resolving multiple hypotheses with minimum traversal of the robot.

A. Approach

In our approach Markov localization is used to find out the pose of the robot, but it is well known that such global localization methods result in conflicting hypotheses of the pose in a symmetric world.

A two staged approach to resolve multiple hypotheses is presented. The approach can quickly zero in on the actual pose of areal robot and resolve hypotheses even in non polygonal environment.

The global pose estimates are computed through the framework of Markov localization and the beliefs updated accordingly [1] For active localization this algorithm works in two stages.

1) Stage 1: We use the concept of virtual robots and sensors. We imagine a robot centered at each of the possible hypothetical poses returned by the global localization algorithm in the form of a multimodal distribution. These are the virtual robots , whose sensors are the virtual sensors. The virtual sensors have an infinite sensing range, unlike real sensors. One of the poses is closest to the actual robot pose. Hence in a symmetric environment if the passive Markov localization returns an m-model distribution of robot beliefs, there are m-virtual robots. Each of these has n virtual sensors, that fire in as many directions. As we know the map of the environment, we can fire virtual sensors to determine the direction for which the maximum value of M is obtained, where M is

$$M = Max(\alpha \times A_i + \beta \times B_i) \tag{1}$$

TABLE I
Computation of A_i and B_i

For m virtual robots and n virtual sensor the following table illustrates how A_i and B_i are computed.

Direction	No. of same reading	No. of different reading(A_i)	Sum of difference in reading (B_i)
1	m_1	m-m_1	$d_1 = \sum_{i,j=1}^{m} \|s_{i,1} - s_{j,1}\|$
2	m_2	m-m_2	$d_1 = \sum_{i,j=1}^{m} \|s_{i,2} - s_{j,2}\|$
.	.	.	.
.	.	.	.
.	.	.	.
n	m_n	m-m_n	$d_1 = \sum_{i,j=1}^{m} \|s_{i,n} - s_{j,n}\|$

Here A_i is number of distinct virtual sensor readings along a particular direction across the m robots. For example in Table 1 along direction 1, if there are m_1 identical readings across the m virtual robot then A_1=$m - m_1$,as there are $m - m_1$ different readings. B_i is the sum of differences in readings across all robots along direction i. In Table 1 $s_{i,j}$ refers to the sensor measurement for robot i along direction j. Choosing max (A_i), we move along the direction that resolves the maximum number of hypotheses. It is evident that, along the direction where the maximum number of virtual sensor readings are different, the robot can be expected to come up with a unique hypothesis easily. Non-uniqueness of hypotheses arises because the robot sees the same in all directions at more than one location in an environment. In other words if the robot sees something unique along a direction that is not seen anywhere else in the environment in principle the robot should obtain a unique hypothesis of its state at that location. Hence more the number of unique (different readings) more is the probability of obtaining a unique hypothesis at that location. In other words in the equation below p(s/l) increases at locations where number of unique readings are more and hence the belief that the robot is in that state $Bel(L_t$=$l)$ becomes singularly high for that location when compared with others after normalization

$$Bel(L_t = l) \leftarrow \frac{p(s|l)\ \widehat{Bel}(L_t = l)}{p(s)} \tag{2}$$

In equation (2) above the notations have the same connotation as those in [1]. Also it is sometimes beneficial to move along the direction where the difference in readings across robots is large. This is monitored by the second term B_i.To avail of the benefit of both A_i and B_i, we combine them using weights α and β. In our simulations we set $\alpha = 0.95$ and $\beta = 0.05$.

We move along the direction for which M is maximum. It is possible that after moving along the direction for the specified distance, the robot would be unable to localize,or that several directions could have the same maximum M. In this case, the next stage of the algorithm comes into play : The distance to be moved along M is given by the distance at which, the real robot sensor, sensing along that direction, will be able to detect the closest obstacle.

2) Stage 2: Assume that we have m virtual robots ,and that each virtual robot i has p frontiers. Each frontier j denoted by f_{ij} where $i \in [1, m]$ and $j \in [1, p]$. It is to be noted that the number of frontiers is the same for all robots, otherwise the pose estimate would be trivial. By frontier, we mean the boundary between the unoccupied and unknown areas [14] in the occupancy grid like sense. Since the frontiers are identical for each of the i virtual robot poses, the subscript i is redundant and removed.

The best frontier capable of eliminating the maximum number of hypotheses, among the f_j frontiers is selected ; let this be denoted by f_s . These are the children of the virtual robot pose. The virtual robot pose forms the root node of the tree and is denoted by f_r. We choose that frontier f_j, $j \in [1, p]$ for which $\sum_{i=1}^{n}(A_i = m - m_i)$ is maximum. If f_s turns out to be a unique robot hypothesis then the algorithm exits. Otherwise, we apply stage 1 of the algorithm at f_s. If stage 1 fails to localize, a new set of child frontiers is generated at f_s. This process repeats as a depth first search from the root node. The search along a particular child (here f_s) of the root terminates when the pose estimate is complete, or there are no more child frontiers of f_s to be visited. The algorithm then branches to another child of f_r and the process is repeated till all hypotheses except one are eliminated.
The steps of the algorithm are given below.

B. Algorithm

1. A set H of possible poses is extracted by a clustering algorithm, from the multimodal distribution of robots' pose beliefs returned by Markov passive localization.
2. If there is only one element in H ,then return the expected value of robot and exit.
3. For all h in H
   Fire the virtual sensor in n directions.
4. For all h in direction i where $i \in [1, n]$,calculate A_i and B_i as detailed before.
5. Calculate $M = Max(\alpha \times A_i + \beta \times B_i)$ for all n; denote the direction corresponding to M by D.
6.If M is obtained for a single direction,
   then move the robot in that direction.
7. If unable to localize after moving along D, or D is not unique, goto step 8.
8. Find the frontier cell f_{ij} for all virtual robots i, and insert these cells in a tree, with the root being the virtual robot pose f_r.
9. For all frontiers f_j of f_r, find the number of different sensor readings in the k^{th} direction for all $1 \leq k \leq n$. Choose the frontier cell f_s that has the maximum number of different sensor readings across all robots and directions.

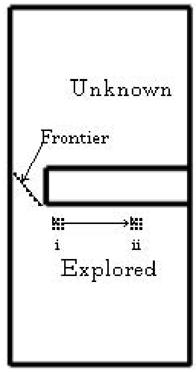


Fig. 1. The robot action of moving towards right from i to ii does not change the frontier grids or result in new frontier grids.

10.Perform a depth first search centered at f_s, by alternating steps 3-6 and 7-9. The cardinality of the hypothesis set H changes at every child selected during the search. A search along a particular child of f_r,f_s stops when there are no more children of f_s or there is only one element in H at a particular child, in which case the algorithm exits.
11.Repeat step 10 for all children ,f_j of f_r, till the termination condition is met.

C. A convergence proof for the active localization algorithm

Claim: The active localization algorithm presented will find its actual pose if there exists a location in the given map where the passive localization module would return a unimodal distribution.

A proof for the above claim is delineated through the following steps. These steps are briefly stated without elaboration to conserve space.
1. A multimodal distribution of robots belief occurs because the sensors see the same in all directions.
2. The robot needs to move to a location where it sees something different than what it is seeing now.
3. This is achieved by extending the robot's vision beyond what is currently seen(accomplished through virtual sensors) and finding a direction of motion that reduces the number of hypotheses.
4. Alternatively, this is best achieved by moving to that frontier capable of reducing the multiple hypotheses to a minimum.
5. An action that does not lead to seeing something different than what has been seen so far does not reduce the number of hypotheses, or the number of modes, in a multimodal distribution.
6. An example of the action described in step 5 is one that does not lead to the addition of an existing frontier gird. For example in figure 1, the robot action of moving towards the right does not change the frontier grids, or the result in new frontier grids.
7. In a bounded environment, frontier exploration guarantees that all parts of the environment will be explored eventually.
8. Step 7 guarantees that the algorithm would halt.

9. Steps 3 and 4 guarantee that a pose that results in a unimodal belief will be found if one exists.
10. The current algorithm makes sure that steps 3 and 4 are always achieved and 5 never occurs. Hence by steps 8 and 9 the convergence of the algorithm is guaranteed.

The gist of the above delineation is as follows. Firstly exploration is a converging process and is well understood. For example in a bounded environment all frontiers get eventually explored. This guarantees the algorithm halts. Secondly the robot needs to move to such a position where it has something new to see from the current location. This happens by choosing an action that leads to addition of new frontier grids. The question is there could be several such actions that lead to seeing of new areas or addition of new frontier grids. But the sum of all the new areas that can be seen is achieved by recursively visiting the frontiers. In other words all the new areas that can be seen as a result of reaching new positions from a given hypothesis location is contained in the new areas seen by moving to frontier locations from that position. Thus frontiers are a sufficient set for discerning an unique hypothesis state. Hence the algorithm would converge to a unique state since frontiers are a sufficient set to discern a new hypothesis and all frontiers can be visited. We admit that this could possibly explained better through figures. But then brevity of space constrains us from doing so. In comparison with the method of choosing random positions we state the following. All the new areas seen from the random positions is eventually contained in the new areas seen from frontiers. Hence frontiers are a sufficient subset of the random positions generated in the UAL method [4] to generate a unique hypothesis. They are fewer in number and easily generated. The tradeoff however is that frontiers are farther away than the average candidate point generated by UAL.

IV. Simulation Result

Figure 2(a) shows the multimodal distribution returned by passive localization algorithm. The thick black dot in the third cluster from the left on the top row is the actual position of the robot and is labeled R. Figure 2(b) illustrates the firing of the virtual sensors, from the various hypothesized positions of the robot or the virtual robots. The direction of motion D resulting from the first stage of the algorithm is shown by a dark line with an arrow for the third virtual robot alone. Evidently this is the direction of motion for the actual robot. The actual robot is not moved immediately; it moves only after the algorithm finds the overall path that results in localization from the application of stages 1 and 2. Figure 2(c) shows an intermediate stage in localization and 2(d) shows the robot having localized by breaking all other ties. In this case stage 2 was not required since the algorithm computed that, by moving along direction D, till the nearest obstacle in that direction, complete localization would occur.

Figure 3 shows active localization for the same initial robot pose in figure 3(a) by method of entropy minimization. Figures 3(b)-3(d) show the robot visiting previous positions as it localizes. This happens because often an earlier visited position becomes the position of maximum information gain at the new location. A possible way to avoid this is to choose the best possible path over several actions instead of choosing the best possible next action. However that makes the search expensive in space and time and even for a single step computed as Table 2 shows, the time taken by entropy minimization is far greater than the present scheme. Table 2 compares the time taken by active localization through entropy minimization and the current algorithm for computing the best action at a given position. As columns 3 and 4 depict, the current algorithm is at least 3 time faster on an average. While the table is shown only for 4 robot positions due to space constraints, similar comparisons were the norm for several positions across several runs. These comparisons were done on a Pentium-3 machine running the Fedora Core 2 version of Linux, with kernel 2.4.10.

TABLE II

COMPARISON BETWEEN METHODS.

S.No.	Position	With entropy (in sec)	Current Algorithm (in sec)
1.	235,255	0.299	0.063
2.	400,225	0.319	0.042
3.	350,250	1.608	0.060
4.	150,250	0.255	0.156

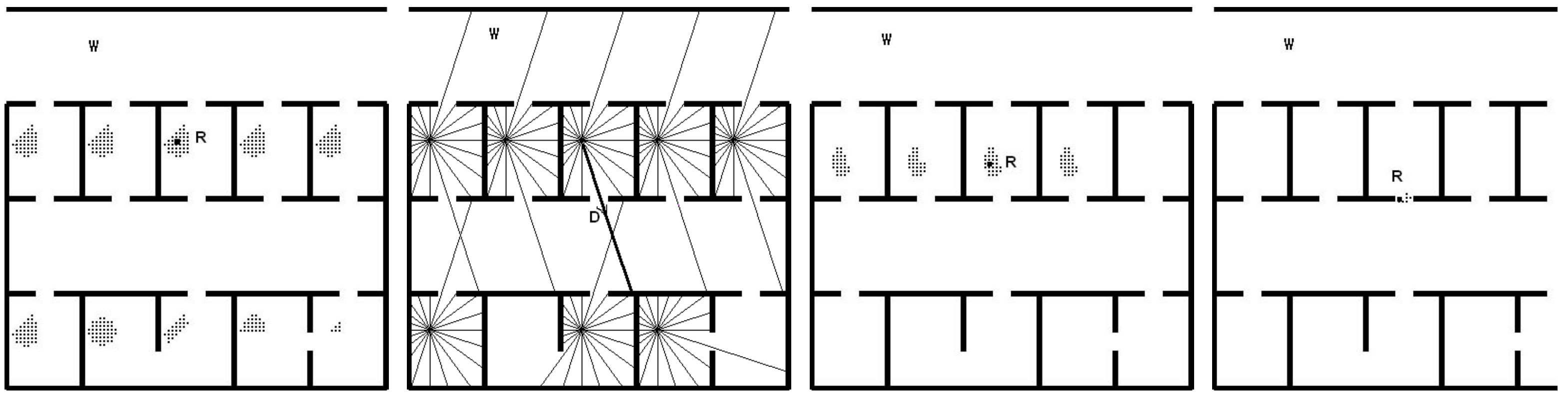
Comparison of time taken by entropy minimization and the current algorithm (columns 3 and 4) for the best action at various robot positions (column 2).

Figure 4 shows, active localization by the current approach, using the second stage. Figure 4(a), shows the initial hypotheses position of the robot (the root of the tree). Among the two frontiers f_1 and f_2, the algorithm chooses f_2. The frontier locations are also shown by frontier grids along the lines representation reported in [14]. From f_2, five more frontiers are formed (which are labeled once again as f_1-f_5) in figure 4(b). Among these frontiers, f_3 was chosen as the frontier capable of eliminating maximum number of hypotheses. This is because the number of sensor readings being identical for the two possible virtual robot positions is the least at f_3 as shown in fig 4(c). A path to f_3 is planned and the robot localized at position P while nearing f_3. In the figure the frontiers are marked for only one of the virtual robot positions, but it is evident that they are the same for all other virtual robot positions.

It is to be noted that both figures 2 and 4 are not simple polygons. Figure 4 is distinctly nonpolygonal due to curvature and figure 2 is not a simple polygon due to the presence of the long wall at top that is disconnected from the rest of environment.This wall is shown labled W in figure 2.

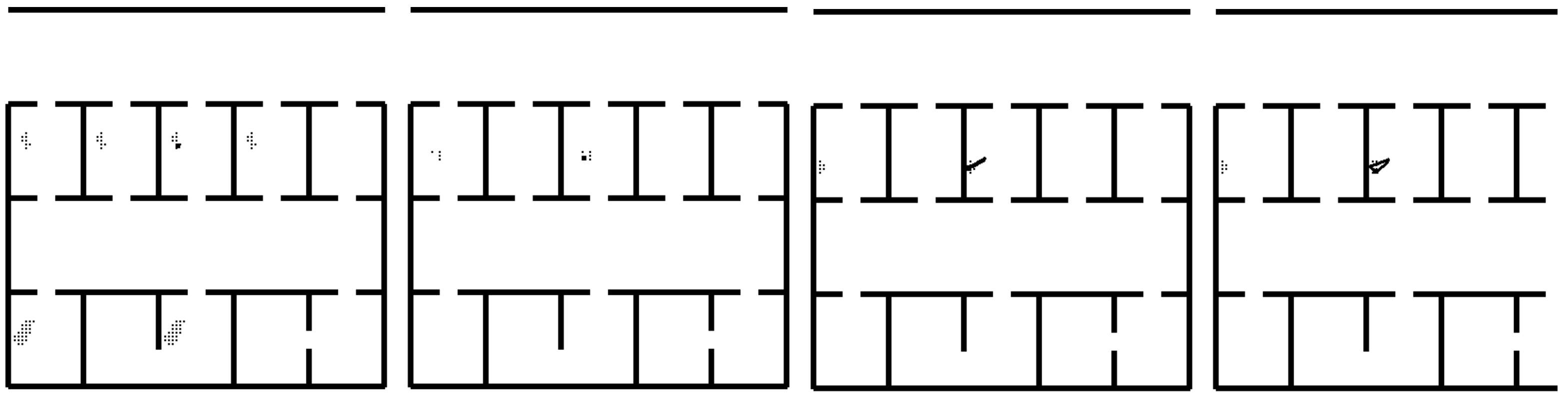
V. Experimental Results

In this section we present the experimental results obtained with Active Media's Amigobot, equipped with 8 range sonars. A symmetric environment was constructed by lining



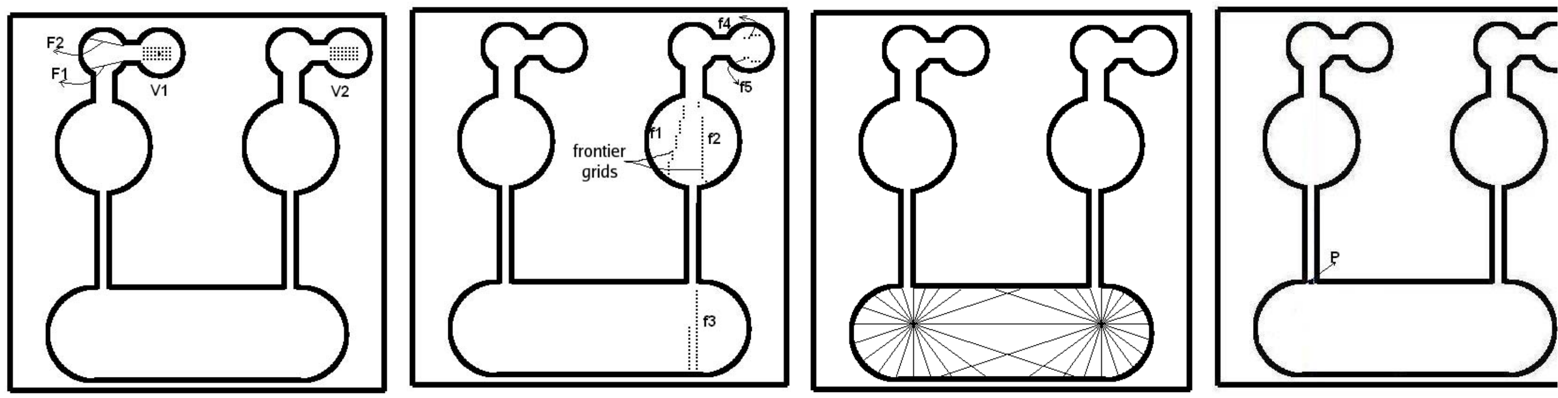
(a) (b) (c) (d)

Fig. 2. 2(a): Multimodal distribution of belief returned by the passive Markov localization. W is a wall at the top disconnected from the rest of the environment. The actual robot position is shown by a thick black dot and labeled R. 2(b): Firing of virtual sensors from the virtual robot position and D represent the best direction for localization is shown by a thick line,with arrow for the 3^{rd} virtual robot pose from the left on the top. 2(c): An intermediate stage where several hypotheses have been eliminated. 2(d): The final localized position.



(a) (b) (c) (d)

Fig. 3. 3(a): Multimodal distribution of belief returned by the passive Markov localization.3(b)-3(d) shows the the robot visiting previous positions as it localizes through entropy minimization.



(a) (b) (c) (d)

Fig. 4. 4(a): Initial hypotheses position of the robot with frontier f_1 and f_2 shown by the frontier grids. 4(b): Next five frontiers f_1-f_5 from f_2. The algorithm selects f_3 4(c): Virtual sensor readings from frontier f_3. 4(d): The final localization on nearing f_3 at location P shown by an arrow.

up cardboards between the walls of a corridor (Fig 5(a)). Figures 5,6 and 7 depicts the various stages of an experimental run where localization was achieved by involving first stage alone. The left half of the figure shows the actual robot in the environment and the right its corresponding position returned by the localization algorithm running on client PC. Figure 5(a)and 5(b) shows the initial pose of the robot and the corresponding hypothesis location calculated by the algorithm. Figures 6(a) and 6(b) depict the directions in which the robot decide to move based on application of stage 1. This is shown by the arrow in figures 6(a) and by a dark line in 6(b). Figures 7(a) and 7(b) show the final localization position of robot.

VI. Conclusion

This paper presents an active localization algorithm based on two stages. The first stage of the algorithm chooses the best possible direction that can that can eliminate the maximum number of hypotheses and the second stage chooses the best frontier to visit that does the same. Simulation and experimental results confirm the efficacy of the algorithm. Comparison with other popular method of active localization clearly point out the benefits of the current method. A future scope of this work is to extend it to a multi-robotic setting.

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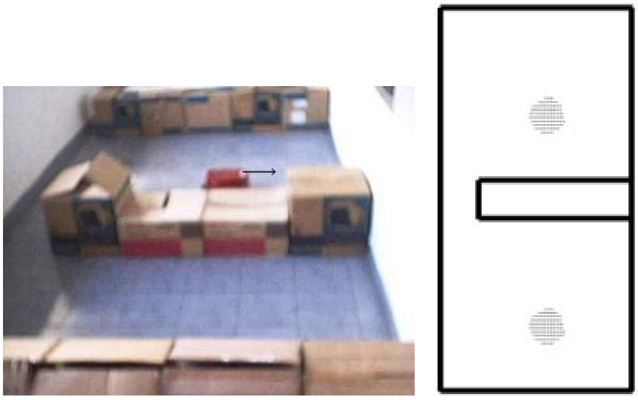
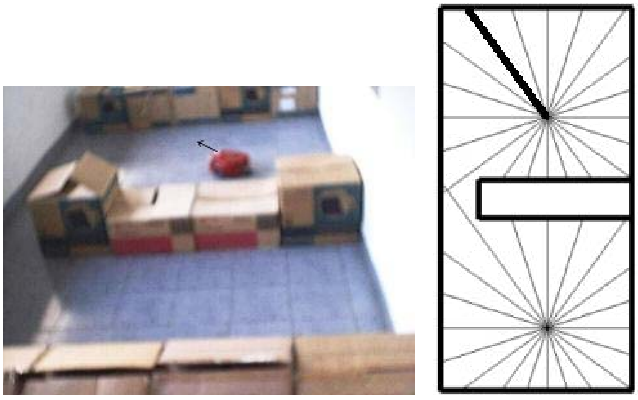


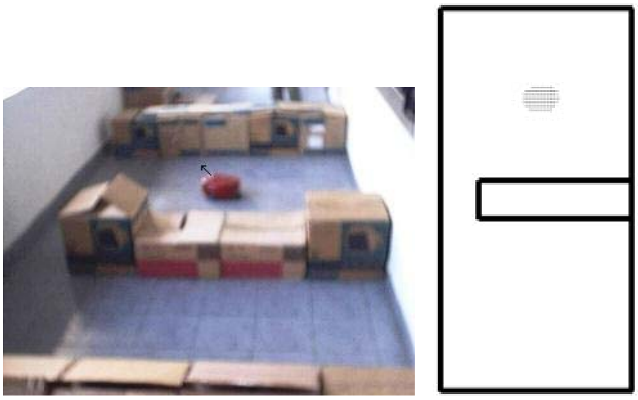
(a) (b)

Fig. 5. Multi modal distribution of belief returned by the passive Markov localization.



(a) (b)

Fig. 6. Choosing direction D for wich M is maximum and move.Shown by arrow in left figure and as thick line in right figure.



(a) (b)

Fig. 7. Final localized position.