An Inquiry Into Modern Cryptography

Kannan Srinathan
IIIT-Hyderabad

What is cryptography about?
Why is cryptography important?
How to solve it (cryptography)?

CRYPTOGRAPHY

Is about `**solving**' **impossible** problems

Cryptography …

… has to brazenly circumvent logical *no-go theorems* !

Sample No-Go*es*

- Illustrating Logical No-Go (Russell's Paradox): Let S be the set of all sets that do not contain itself. Does S belong to S?

  Ans: Yes *and* No!

1. Should the machine know your *password*?

   Ans: Yes (for checking) *and* No (for secrecy)

2. Can you spend your digital cash?

   Ans: Yes (the original) *and* No (the copies)

3. Should there be CCTV cameras?

   Ans: Yes (for policing) *and* No (for privacy)

Cryptography is *Fascinating*

- Because …

… no other field of science has so *pleasingly succeeded* in circumventing logical no-go results …

Sample "Successes" against Logical Impossibilities

1. Authenticity with Anonymity!
2. Blinding but Binding!
3. Compression without Collision!
4. Privacy Preserving Personalization!

Cryptography

Is therefore **fundamental**

Cryptography is *Fundamental*

- Because …

… it has extended its success story by circumventing logical no-go theorems in ***other areas*** too …

(S)ample Technical Benefits of Cryptography

- Coding Theory
  - Detecting 100% Adversarial Noise
- Distributed Computing
  - Fault -Tolerant Agreement
- Mathematics
  - What is a Proof?: Zero-Knowledge Proof Systems
- Algorithms
  - Pseudorandomness and Derandomization

Rest of the talk …

How To Solve It?

… the power of adversarial interference

The Cryptographic Method

- Understand the (original) impossibility
- Bring in another impossibility
  - In just about the correct proportion
- Make the impossibilities destructively interfere each other
  - … to make a solution possible!

Adversarial Interference (has happened before crypto too)

- Randomized Algorithms
- Game Theory and Byzantium

Some Famous Adversities
(that enable cryptography)

- **Computational Adversity**
  - **Eg. Limited resources**
- **Physical Adversity**
  - **Eg. Quantum and Relativistic Mechanics**
- **Practical Adversity**
  - **Eg. Scheduling and Software Bugs**
- **Philosophical Adversity**
  - **Eg. Clash of Fundamental Definitions**

We'll See One Example For Each Kind of Adversarial Interference

Four examples in all

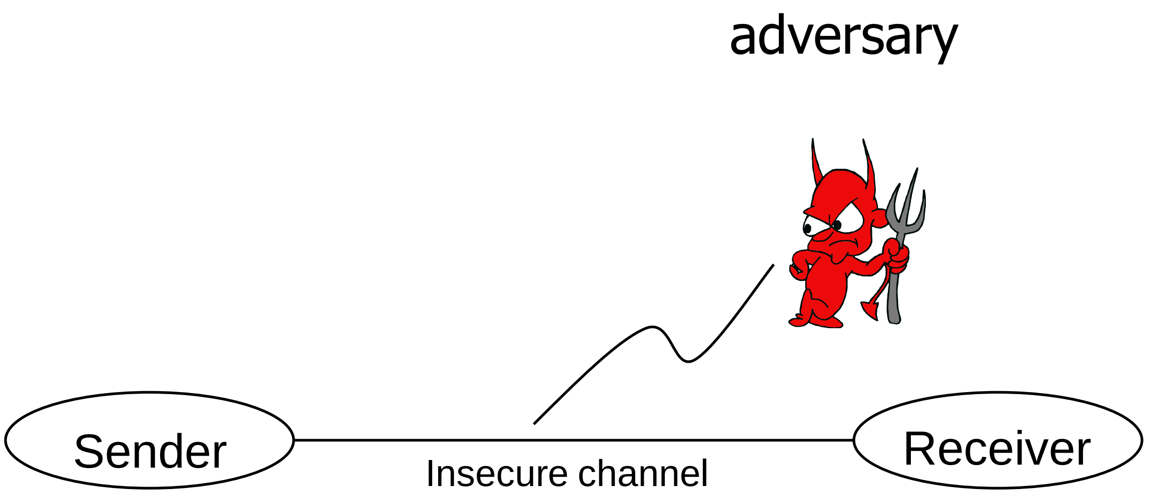
Our First Example

Is Secure Communication a Cryptographic Problem?

Yes! It is a Logical No-Go!

Why?

Secure Communication is Impossible!



- At time t_0
  - Information@Receiver = Information@Adversary

Recall: Kerckhoff's Principle

- At every subsequent instant of time
  - Information gained by receiver = Information gained by adversary

How to Circumvent the Impossibility?

Only Two Ways

At time t_0

Information@Receiver

is (perceived as) greater than

Information@Adversary

OR

At some subsequent instant of time

Information gained by receiver

is (perceived as) greater than

Information gained by adversary

The First Way …

Representation matters, indeed!

Natural Numbers, Efficiency of Operations and Modern Cryptography

Ease of Computation Depends on the Representation

It also depends on the operation!

Ease/Speed of Operation Depends on The Representation

- viii * xvi = cxxviii
- 8 * 16 = 128
- $2^3 * 2^4 = 2^7$

- viii + xvi = xxiv
- 8 + 16 = 24
- $2^3 + 2^4 = 2^3.3$

- viii < ix is true
- 8 < 9 is true
- $2^3 < 3^2$ is true

Top Three Most Frequent Operations

- Addition (+)
- Comparison (<)
- Multiplication (*)

Why is the Decimal System Popular?

	Addition	Multiplication	Comparison
ROMAN	SLOW	SLOW	SLOW
DECIMAL	FAST	MEDIUM	FAST
PRIME PRODUCT	SLOW	FAST	SLOW
RESIDUE SYSTEM	FAST	FAST	MEDIUM

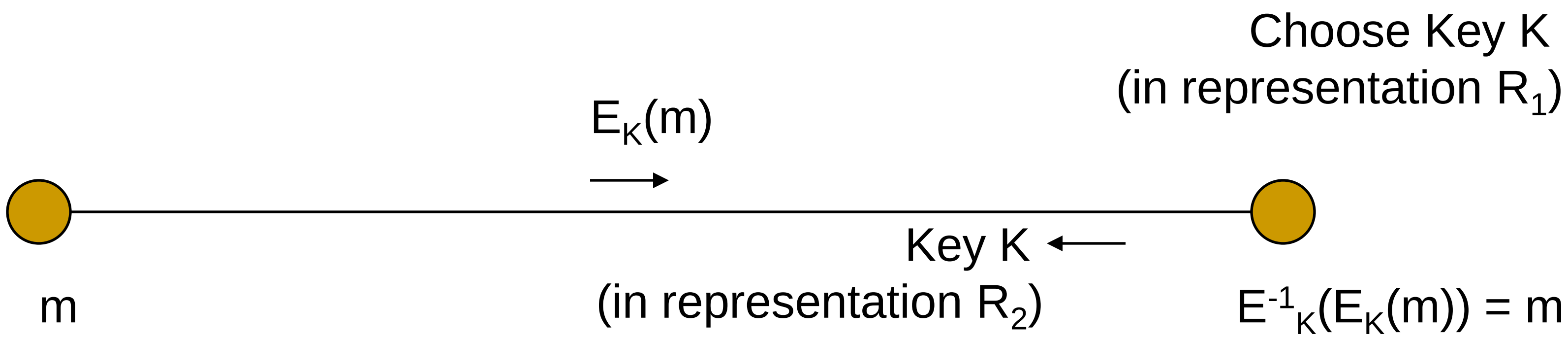
Is There a Representation Where all Common Operations are FAST?

Not Easy!

Slowness is ADVANTAGEOUS too!

Public Key Cryptography

Secure Communication



In Representation R_2

- **Operation E_K is FAST**
- **Operation E_K^{-1} is VERY SLOW**

In Representation R_1

- **Operation E_K^{-1} is FAST**

EXAMPLE RSA Cryptosystem

R_1 : Product of Primes

R_2 : Decimal

E_K : Modular Exponentiation
m^e mod K

RECALL: How to Circumvent the Impossibility?

At time t_0

Information@Receiver

is (perceived as) greater than

Information@Adversary

OR

At some subsequent instant of time

Information gained by receiver

is (perceived as) greater than

Information gained by adversary

Our second example

Secure Communication in Quantum Channels

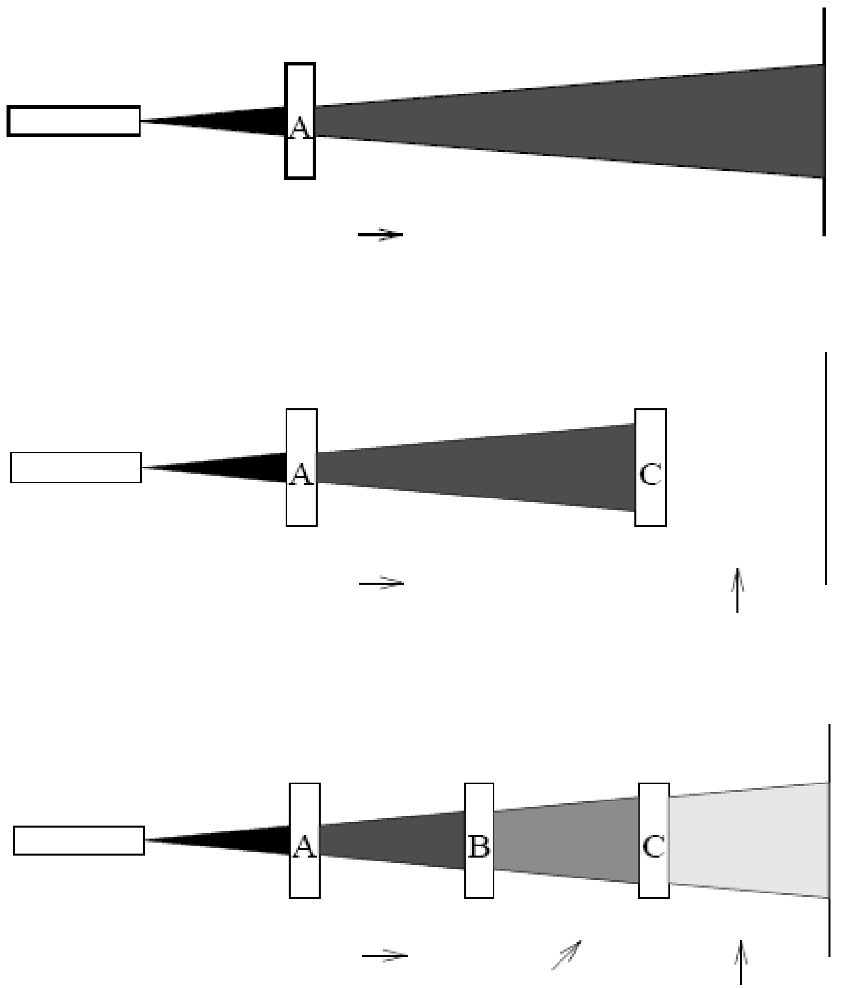
Natural Adversary

Quantum World: It’s *Bizarre!*

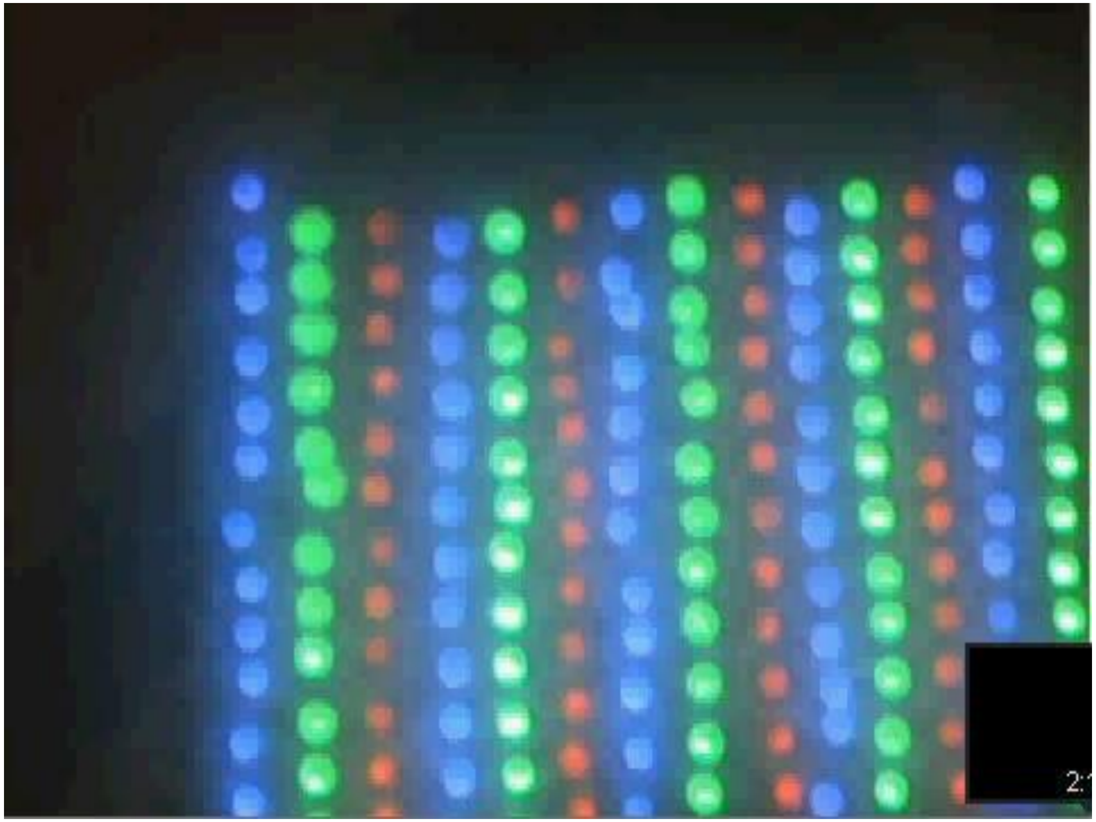
An Experiment with Photons

The Three Polarizers

The Photon Experiment



The Photon Experiment (Contd.)



Qubits

An Explanation

Qubits

- A quantum bit, or qubit, is a unit vector in a two dimensional complex vector space for which a particular basis has been fixed and is denoted by:

$$\{|0\rangle, |1\rangle\}$$

- Qubits can be in a superposition of $|0\rangle$ and $|1\rangle$ such as

$$a|0\rangle + b|1\rangle$$

where a and b are complex numbers such that $|a|^2 + |b|^2 = 1$.

Measuring a Qubit in the Basis

For the qubit

$$a|0\rangle + b|1\rangle$$

the probability that the measured value is $|0\rangle$ is

$$|a|^2$$

after which the state collapses to $|0\rangle$ and

the probability that the measured value is $|1\rangle$ is

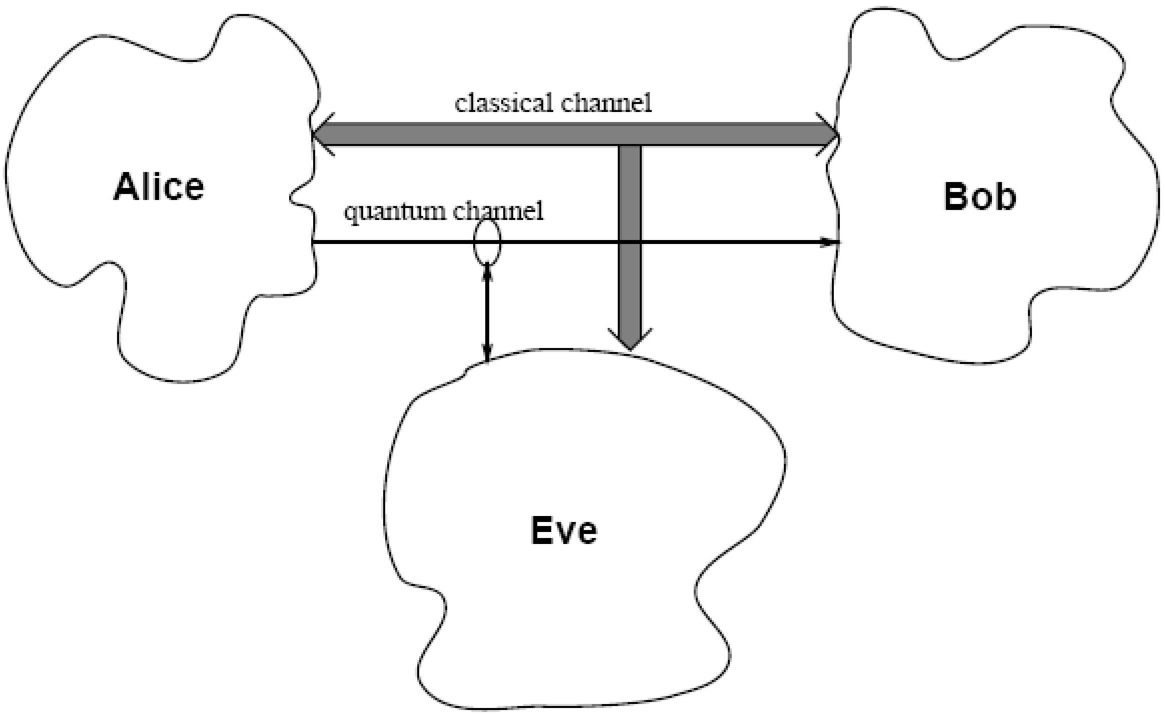
$$|b|^2$$

after which the state collapses to $|1\rangle$

Qubit Model Correctly Predicts the Outcome of Photon Experiment

and several other experiments too!

Quantum Secret Key Establishment Protocol



The Standard Setting

Quantum Secret Key Establishment Protocol

- Two bases are used, say b_1 and b_2

.

- S chooses a random base, and based on the bit to send, it sends a qubit prepared in the corresponding state.

- R measures the qubit received, with a random base. If the base is different from what S used, the bit is lost, else R measures the actual bit (always so, only if an eavesdropper is *absent!*).

Bit	0	1
b_1	↑	→
b_2	↗	↘

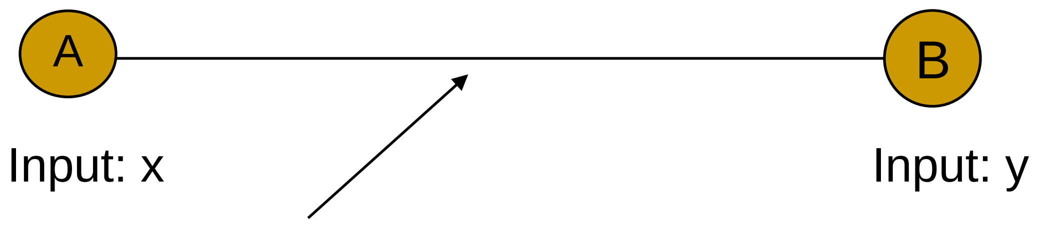
Our third example

Secure Communication in Noisy Channels

Practical Adversary

Secure AND

- Securely Computing $x \wedge y$ in GF(2)



Noise: Any 1 bit out of every block of 4 bits sent will be toggled

Fact: Perfectly Secure AND is impossible in a noiseless channel

Protocol for Secure AND

- A chooses four random bits, r_0,r_1,r_2,r_3 and sends them to B, who receives s_0,s_1,s_2,s_3
  - One of the r_i is different from s_i
  - Three of the others are equal
- A and B compute the following 3-tuples respectively

M =

0	0	0
0	1	0
1	0	0
1	1	1

• A (respectively B) multiplies the i^{th} row of matrix M with r_i (respectively s_i) to obtain a matrix M^A (resp. M^B)

• A (resp. B) adds up the resultant 4 by 3 matrix M^A (resp. M^B) column-wise to obtain a 3-tuple $T^A = (a_0, b_0, c_0)$ (resp. $T^B = (a_1,b_1,c_1)$)

Our Last Example

Philosophical adversity

Some Important Philosophical Questions

- Who is ***honest***?
- How can a software be at ***fault***?
- What is a ***proof***?
- What is ***efficiency***?
- What is **intelligence**?
- What is **security**?

Can a cluster of insecure systems-simulate security?

Welcome to **blockchain**!

Concluding Remarks

Adversarial interference is the key!

Thank You

Questions?