# Introduction to Financial Instruments & Portfolios

Sukrit Mittal

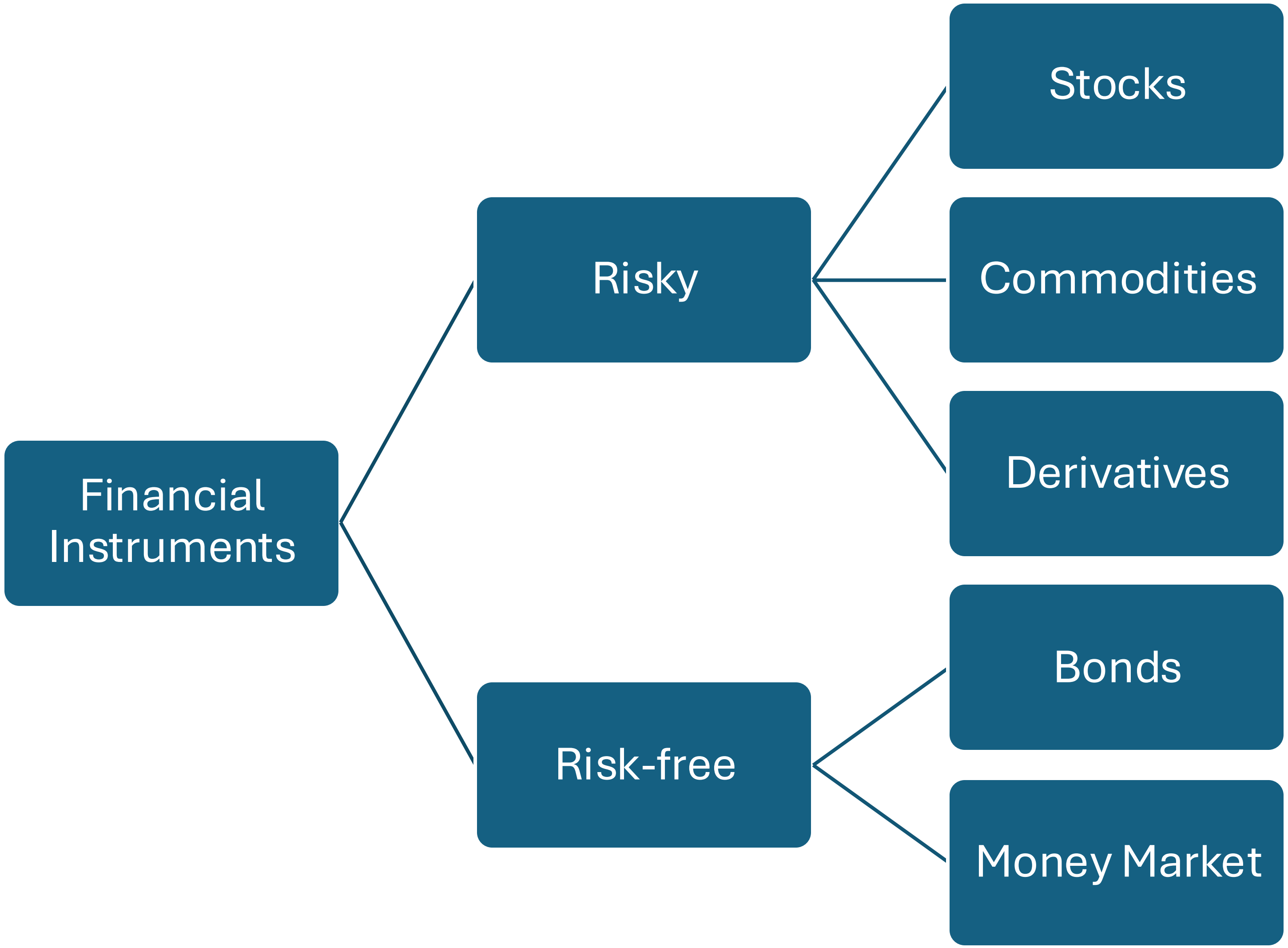
**Session goals:**

- Understand key financial instruments
- Understand No-arbitrage principal
- Intuition for forward and options

# Session Agenda

- Types of financial instruments
- No-arbitrage principal
- Risk & return
- Forward contracts
- Options pricing & risk

# Types of Financial Instruments




A stock represents partial ownership in a company and gives you a claim on its profits and assets.

Commodities are basic physical goods like oil, gold, or wheat that are traded in standardized contracts on markets.

Derivatives are contracts whose value is derived from the price or level of some underlying asset, rate, or index.

A bond is a loan you give to a government or company in return for fixed interest payments and repayment of principal at maturity.

The money market is where very short-term, low-risk debt instruments (like T-bills and deposits) are traded as cash equivalents.

# Risky & Risk-free Assets

**Risky**

- Can be specified as the number of shares of stock held by an investor.
- Current price $S(0)$ is known.
- Future price $S(T)$ is uncertain, it may go up as well as down.

$$\text{Return, } K_S = \frac{S(T) - S(0)}{S(0)}$$

**Risk-free**

- Can be specified as the amount held in a bank account or number of bonds.
- Current price $A(0)$ is known.
- Future price $A(T)$ is also known, as guaranteed by the institution.

$$\text{Return, } K_A = \frac{A(T) - A(0)}{A(0)}$$

# Portfolio

- The total wealth of an investor holding $x$ stock shares and $y$ bonds at time $t = 0, T$ is:

$$V(t) = xS(t) + yA(t)$$

- The pair $(x, y)$ is called a portfolio, and $V(t)$ is called the portfolio value or wealth of the investor at time $t$.

- The price movement between $t = 0$ and $t = T$ leads to a change in portfolio value:

$$V(T) - V(0) = x(S(T) - S(0)) + y(A(T) - A(0))$$

- The portfolio return can be similarly defined as:

$$K_V = \frac{V(T) - V(0)}{V(0)}$$

# Some More Terms

- **Divisibility** is referred to the fact that one can hold fraction of a share or bond.
  - Almost perfect divisibility can be achieved in real world when volume of prices is large compared to unit prices.

- **Liquidity** is referred to the fact that no bounds are imposed on $x$ and $y$.
  - Any asset can be bought of sold on demand at the market price in arbitrary quantities.

- **Long position** is the state of an investor if the number of securities of a particular kind held is positive.
  - Otherwise, we say a **short position** has been taken or the asset is shorted.

# Shorting an Asset

*Can you make profit if a stock price goes down?*

- Borrow shares of a stock (from your broker).
- Sell those shares in the market at the current price.
- Later, buy them back (cover your short).
- Return the shares to the lender.
  - If the price **drops**, you pocket the difference.
  - If the price **rises**, you take a loss because you'll have to buy back at a higher price.

# Shorting an Asset

**Terms & conditions:**

- The owner of the stock keeps all the right to it.
- They are entitled to receive any dividends due and may wish to sell the stock at any time.
- The investor must have sufficient resources to fulfill the resulting obligations.

***Shorting*** can be represented in the portfolio equation using a negative value for either or both of $x$ and $y$. For example,

$$V(t) = -xS(t) - yA(t) + C(t) \geq 0$$

where $C(t)$ is a cash reserve or other bonds submitted as a guarantee. The $\geq 0$ component ensures the guarantee is sufficient.

# Arbitrage

What is **Arbitrage**? – it means taking advantage of price differences of the same asset (or equivalent assets) in different markets, with no risk and no net investment, to lock in a profit.

**Classic example:**

- Gold is trading at 60,000 per 10g in Delhi.
- The same gold is trading at 60,300 in Mumbai.
- You buy in Delhi and simultaneously sell in Mumbai, pocketing the 300 difference per 10g.

**Key points:**

- Simultaneous transactions (buy low, sell high at the same instant).
- No risk (in theory, because the trades cancel each other out).
- No net capital (you're not speculating, just exploiting a mispricing).

# No-Arbitrage Principal

*We shall assume that the market does not allow risk-free profits with no investment.*

Then how does it happen?

*It happens when some market participants make a mistake.*

**Example**: Suppose a dealer in Mumbai offers to buy USD at a rate of INR 85, while there is a dealer in New York ready to sell at a rate of INR 86. – **No direct arbitrage opportunity**

Suppose further that USD can be borrowed at an annual rate of 4% and INR can be invested in a bank at 7%.

- A borrowed principal of USD 10,000 can be converted to INR 850,000. The 7% interest for a year will amount to INR 59,000, leading to a net capital of INR 909,500.
- Converting back leads to USD 10,576. With a loan interest of USD 400, the net profit will be USD 176. – **Guaranteed**

Can you identify the flaw in this example?

# One-step Binomial: Assumptions

- The stock price $S(T)$ can take only two different values in the future.
- Despite its simplicity, it would be able to give you the flavor.

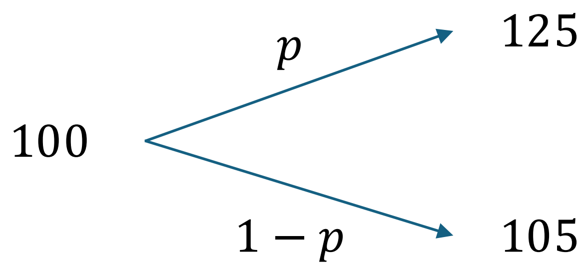
# Example

Suppose that $S(0) = 100$ dollars and $S(T)$ can take two values,

$$S(T) = \begin{cases} 125 & \text{with probability } p, \\ 105 & \text{with probability } 1 - p, \end{cases}$$

Where $0 < p < 1$, and the bond prices are $A(0) = 100$ and $A(T) = 110$ dollar. Hence,

- The return on the stock $K_S$ will be $25\%$ if it goes up, and $5\%$ if it goes down,
- The risk-free return will be $K_A = 10\%$.



# General Representation

One-step Binomial tree:

$$S(T) = \begin{cases} S^u(T) & \text{with probability } p, \\ S^d(T) & \text{with probability } 1 - p, \end{cases}$$

Where $0 < p < 1$, and $S^d(T) < S^u(T)$.

*The choice of stock and bond prices in a binomial model is constrained by the No-arbitrage principle.*

# Proposition

The following restriction must be imposed or else an arbitrage opportunity will arise.

$$\frac{S^d(T)}{S(0)} < \frac{A(T)}{A(0)} < \frac{S^u(T)}{S(0)}$$

We will prove this using negation, building the following cases.

$$\frac{A(T)}{A(0)} \leq \frac{S^d(T)}{S(0)} \qquad\qquad \frac{A(T)}{A(0)} \geq \frac{S^u(T)}{S(0)}$$

# Case 1

Suppose $\frac{A(T)}{A(0)} \leq \frac{S^d(T)}{S(0)}$. In this case, at $t = 0$:

- Borrow the amount $S(0)$ risk free;
- Buy one share of stock for $S(0)$.

This way, the portfolio $(x, y)$ will have:

$$x = 1; \quad y = -\frac{S(0)}{A(0)}; \quad V(0) = 0$$

At time $T$, the value will become:

$$V(T) = \begin{cases} S^u(T) - \frac{S(0)}{A(0)} A(T) & \text{if stock goes up,} \\ S^d(T) - \frac{S(0)}{A(0)} A(T) & \text{if stock goes down.} \end{cases}$$

$V(T) \geq 0$ implies there is an arbitrage opportunity, violating the no-arbitrage principle.

# Case 2

Suppose $\frac{A(T)}{A(0)} \geq \frac{S^u(T)}{S(0)}$. In this case, at $t = 0$:

- Sell short one share for $S(0)$;
- Invest the amount $S(0)$ risk free.

This way, the portfolio $(x, y)$ will have:

$$x = -1; \quad y = \frac{S(0)}{A(0)}; \quad V(0) = 0$$

At time $T$, the value will become:

$$V(T) = \begin{cases} -S^u(T) + \frac{S(0)}{A(0)} A(T) & \text{if stock goes up,} \\ -S^d(T) + \frac{S(0)}{A(0)} A(T) & \text{if stock goes down.} \end{cases}$$

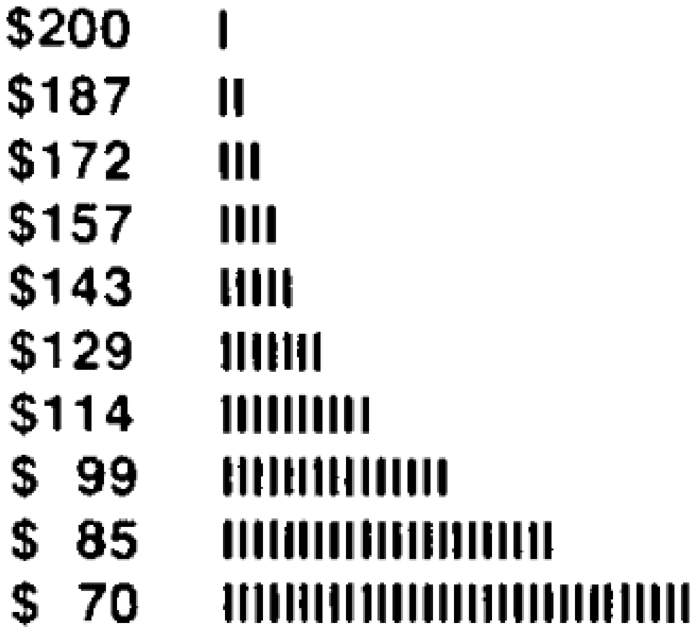
$V(T) \geq 0$ implies there is an arbitrage opportunity, violating the no-arbitrage principle.

# No-Arbitrage Principal

- Modern arbitrage is usually done by algorithms in milliseconds, across currencies, stocks, futures, options, commodities, even crypto.
- The opportunities are small and vanish fast.
- In theory, arbitrage keeps markets "honest" – if one market misprices an asset, **arbitrageurs** rush in and push prices back in line.
- Due to their rate and short-lived nature, the exclusion of arbitrage from our financial models is close enough to reality.

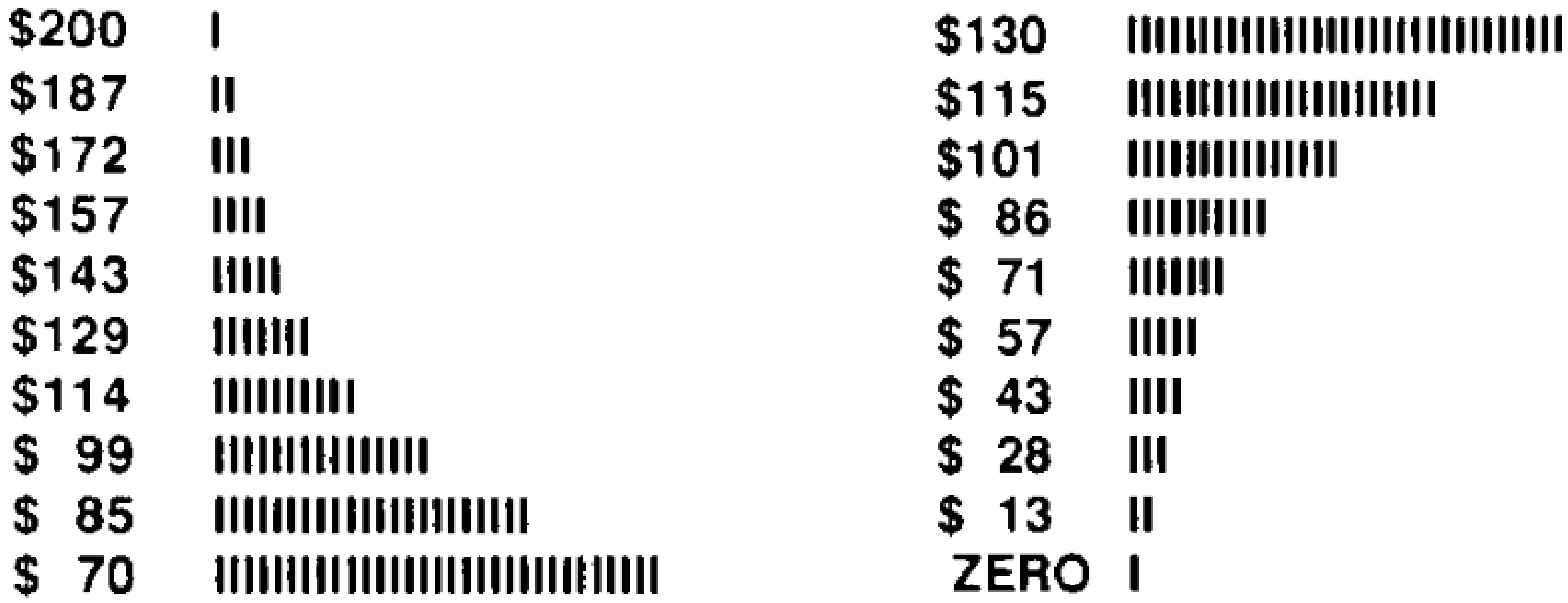
*Mathematically, there is no portfolio* $(x, y)$ *with initial value* $V(0) = 0$ *such that* $V(T) \geq 0$ *with probability* $1$ *and* $V(T) > 0$ *with non-zero probability.*

# Understanding Risk



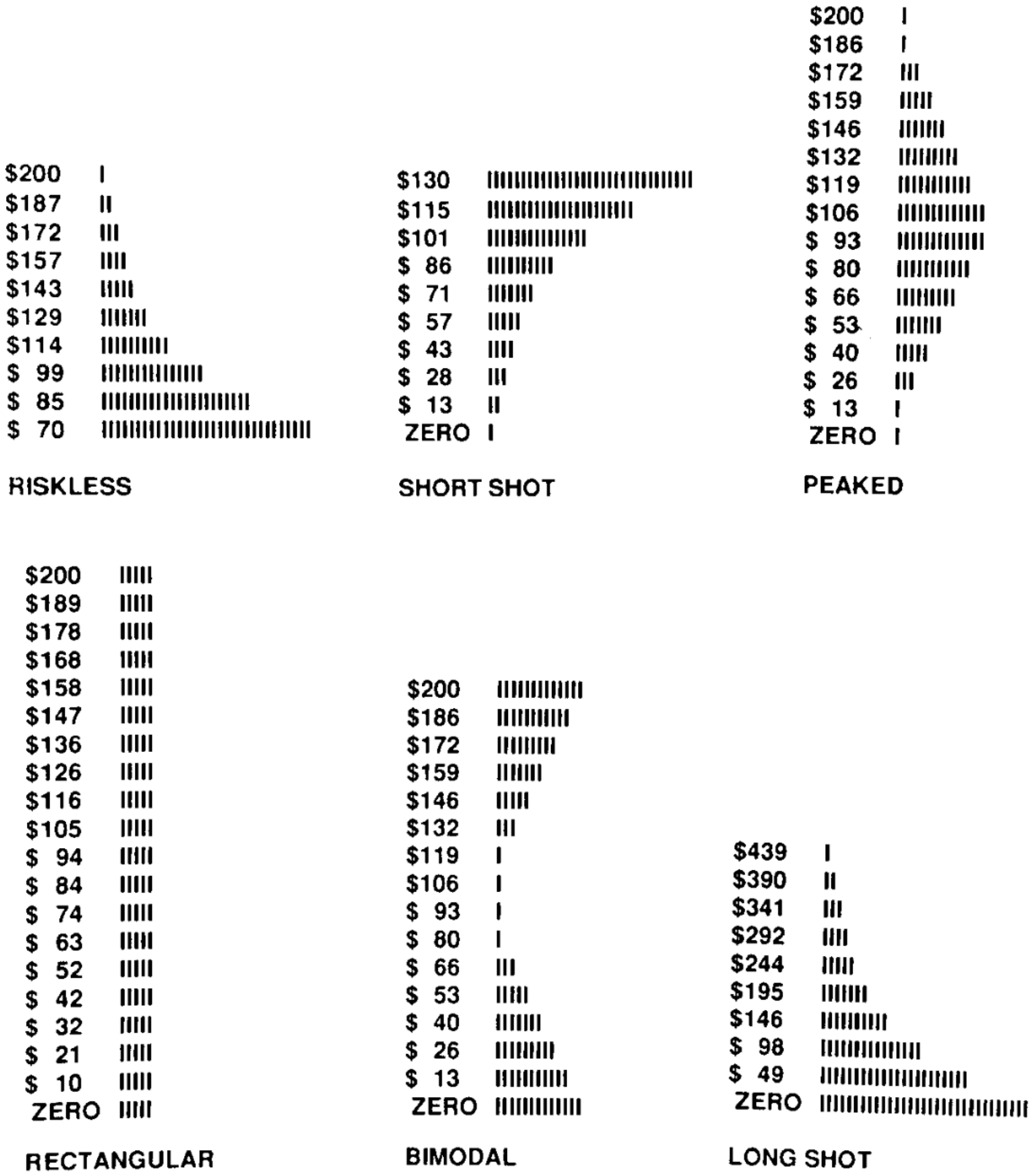

If you have $100, would you invest in this lottery?

# Understanding Risk




If you have $100, which lottery would you invest in?

# Understanding Risk



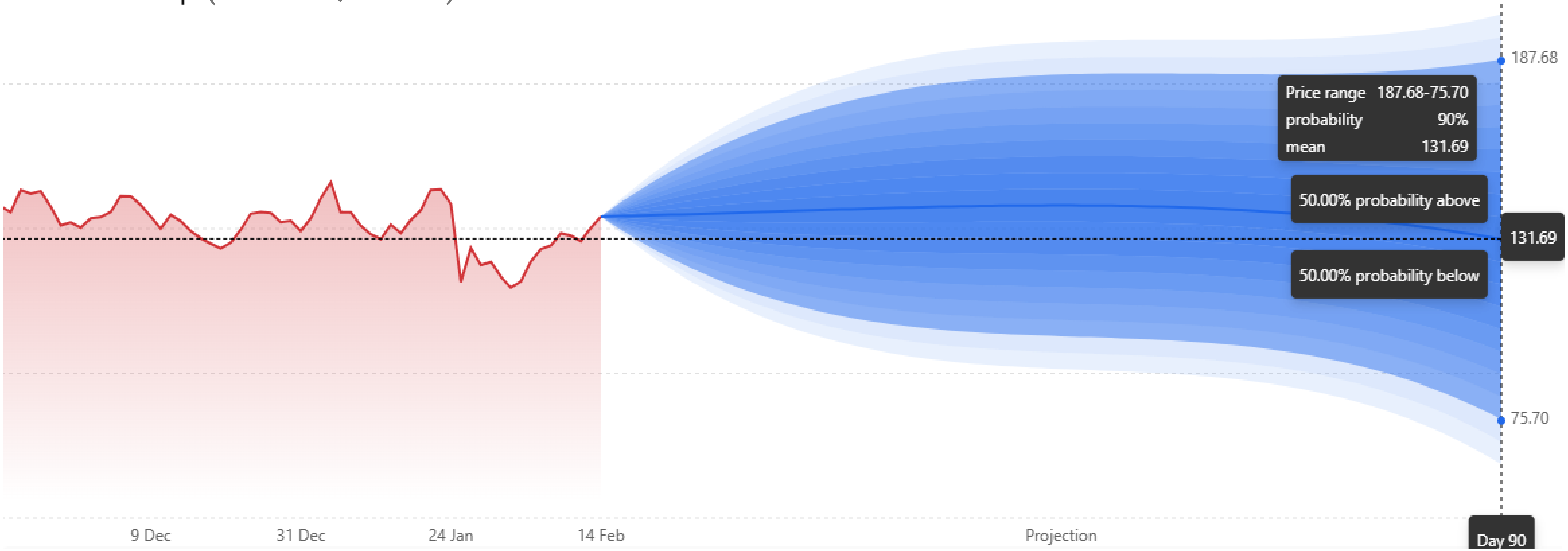

If you have $100, which lottery would you invest in?

riskless > short shot > peaked > rectangular > bimodal > long shot.

# Understanding Risk

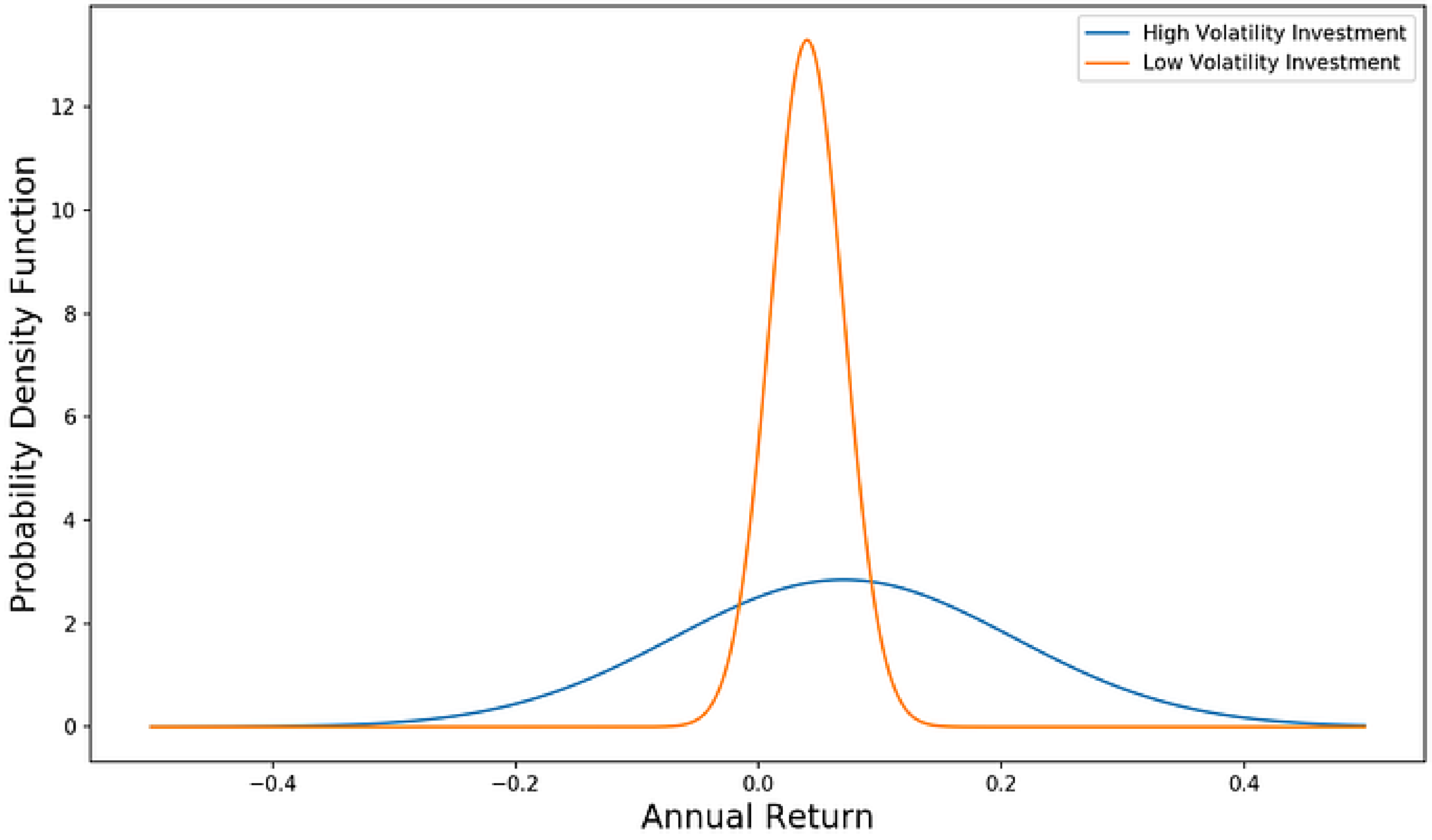
# Understanding Risk

NVIDIA Corp (NASDAQ: NVDA)



# Risk & Return

Assuming normal distribution for stock returns.



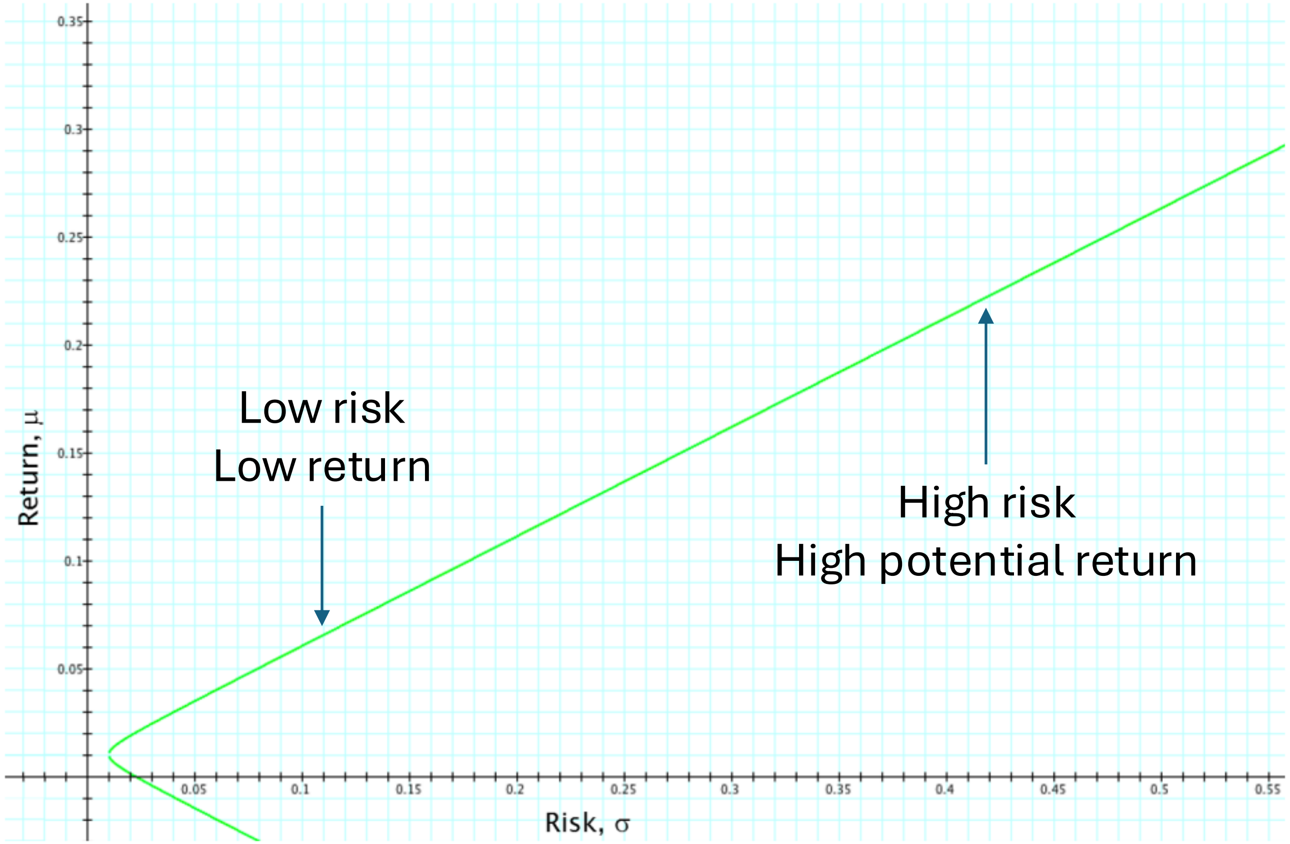
# Risk = Uncertainty of returns

- Investors prefer more predictable outcomes.
- Risk must be quantified to compare different investments.
- Standard deviation is the most commonly used measure,

$$\sigma(K) = \sqrt{\mathbb{E}[(K-\mu)^2]}$$

Where $\mu$ is the expected return.

# Risk/return Tradeoff



# Risk-return

Let $A(0) = 100$ and $A(T) = 110$ dollars, and $S(0) = 80$ dollars.

$$S(T) = \begin{cases} 100 & \text{with probability } 0.8 \\ 60 & \text{with probability } 0.2 \end{cases}$$

Suppose you have \$10,000 and you invest $x = 50$ shares and $y = 60$ bonds. Then,

$$V(T) = \begin{cases} 11{,}600 & \text{if stock goes up,} \\ 9{,}600 & \text{if stock goes down.} \end{cases}$$

$$K_V = \begin{cases} 16\% & \text{if stock goes up,} \\ -4\% & \text{if stock goes down.} \end{cases}$$

# Risk-return

The *expected return* is the mathematical expectation of the return on the portfolio, given by

$$E(K_V) = 16\% \times 0.8 - 4\% \times 0.2 = 12\%.$$

The *risk* of this investment is defined as the standard deviation of $K_V$:

$$\sigma_V = \sqrt{(16\% - 12\%)^2 \times 0.8 + (-4\% - 12\%)^2 \times 0.2} = 8\%$$

Is this a good portfolio?

Invest all the money in bonds.
$K_A = 10\%$
$\sigma_A = 0\%$

Invest all the money in stocks.
$E(K_S) = 25\% \times 0.8 - 25\% \times 0.2 = 15\%$
$\sigma_S = 20\%$

# Forward Contracts

**Forward contract** is an agreement to buy or sell a risky asset at a specified future time, known as the **delivery date**, for a price $F$ fixed at the present moment, called the **forward price**.

- An investor who agrees to buy the asset is said to *enter into a long forward contract or to take a long forward position*.
- An investor who agrees to sell the asset is said to *enter into a short forward contract or to take a short forward position*.
- No upfront payment.

# Options

- **Option** is a financial contract giving the right but not the obligation to buy or sell an asset.
  - **Call** Option: right to buy at strike $X$.
  - **Put** Option: right to sell at strike $X$.
- A premium is paid today to receive this flexibility.

- Payoff formulas:
  - **Call**: $\text{Payoff} = \max(S(T) - X, 0)$
  - **Put**: $\text{Payoff} = \max(X - S(T), 0)$

# Options Pricing

Let us change the portfolio representation to: $(x, y, z)$

$$V(t) = xS(t) + yA(t) + zC(t)$$

Pricing the option is equivalent to identifying $C(0)$.

$$S(T) = \begin{cases} 120 & \text{with probability } p \\ 80 & \text{with probability } 1 - p \end{cases}$$

Call option:

$$C(T) = \begin{cases} 20 & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}$$

Assume $A(0) = 100$ and $A(1) = 110$.

# Replicating the Option

$$xS(T) + yA(T) = \begin{cases} x120 + y110, & \text{if stock goes up} \\ x80 + y110, & \text{if stock goes down} \end{cases}$$

Hence,

$$C(T) = \begin{cases} x120 + y110 = 20 \\ x80 + y110 = 0 \end{cases}$$

Solving these equations give: $x = \frac{1}{2}; y = -\frac{4}{11}$

This means that to replicate this option, we need to buy ½ stock and take a short position on -4/11 bonds (borrow $400/11 in cash).

$$C(0) = \frac{1}{2} \times 100 - \frac{4}{11} \times 100 = 13.64$$

# Managing Risk with Options

- If I have \$1000:
  - Becomes \$1200 if the stock goes up.
  - Otherwise, \$800.
- Instead, I can buy 73.33 options priced at \$13.64 each.
  - Becomes \$1466 if the stock goes up.
  - Otherwise, 0.
- Purchasing an option clearly seems riskier.

- Alternatively, assume a risky asset:
  - $S(T) = \begin{cases} 160 \\ 40 \end{cases}$
- Call option price: \$31.81, which needs to be repaid as \$35 (it's a loan).
- Borrow the money, purchase the option. Repay the loan at time $T$.
- Hence, $S(T) - C(T) + 35 = \begin{cases} 135 \\ 75 \end{cases}$
- Clearly, the risk is reduced.

# Thank you!