Bessel Transform for Image Resizing

Ganga Mohan P
Department of Electrical Engineering
Blekinge Institute of Technology
Karlskrona, Sweden
ganga39@gmail.com

Chetana Prakesh, Surayakanth V. Gangashetty
Speech and Vision Laboratory
International Institute of Information Technology,
Hyderabad, India
chetana@research.iiit.ac.in, svg@iiit.ac.in

Abstract **— In many circumstances in image processing, image resizing is done, either to magnify or to reduce the size of a digital image. The spatial domain based resizing methods such as bilinear interpolation and bi-cubic interpolation are simple and work better for image size magnification. The main drawback in using them is that they are not suitable for image size reduction. In this paper, we propose a new method for image resizing based on Bessel transform (BT). The performance of image resizing based on BT is compared to that of spatial domain based resizing techniques, the results are viewed in terms of Peak Signal to Noise ratio (PSNR) and Mean Square Error (MSE). Experimental results confirm that the proposed method maintains better image quality when image size is enlarged and also when image size is reduced.**

Index Terms: **Image Resize, Bessel Transformation, Bilinear interpolation, Bi-cubic interpolation**.

I. INTRODUCTION

The importance of image resizing scheme is greatly felt in the fields of medical image processing, computer graphics, image database, etc. The problem mainly occurs when the user wants to display an image at different resolutions depending upon the resolution of the display device [1]. Presently in image processing, we often need to perform zoom in, zoom out and crop operations on huge quantity of digital images and all these operations involve image resizing [2].

The goal of image resizing is to magnify or reduce the size of the image and simultaneously preserve the details and perceptual quality of the original image. One simple method of image resizing is the pixel replication method which is based on nearest neighbor interpolation. More sophisticated methods like bilinear interpolation [3, 4] and bi-cubic interpolation [5, 6, 7] maintain better image quality than the pixel replication method when resizing is done to enlarge an image. But these methods are not entirely suitable for image reduction because of potential aliasing problems [8]. Most of the image resizing algorithms work based on the principle of spatial interpolation and are not adapted to the content of the image. They may fail to preserve the relevant features of an image, especially during image reduction [9]. Other resizing techniques are more promising, but also have their own drawbacks.

As none of the image resizing techniques are expected to work well for all types of images and different methods work well on different types of images, we need to select the method based on the application. This prompted us to investigate the use of Bessel Transform (BT) in image resizing. BT has been used as a technique for feature extraction of speech signals used for various speech signal processing applications [10]. This paper highlights the performance of BT based method over bilinear interpolation and bi-cubic interpolation methods.

The paper is organized as follows: in Section II, we describe the mathematical formulation of 2-D BT. In Section III, we describe the method of image resizing using BT. The various image quality metrics are discussed in Section IV. In Section V, we discuss the results of image resizing. In section VI, we give a summary of the study.

II. BESSEL TRANSFORM

The use of 1-D BT for processing 1-D signals is described in [11]. In this section we describe the 2-D BT. A finite duration signal $x(t_1,t_2)$ in the interval $0 \leq t_1 \leq a, 0 \leq t_2 \leq b$, is represented by using the zero order Bessel functions of the first kind in an infinite Bessel series as,

$$x(t_1,t_2) = \sum_{m_1=1}^{a}\sum_{m_2=1}^{b} c(m_1,m_2) J_0\left(\frac{\alpha_{m_1}}{a}t_1\right) J_0\left(\frac{\alpha_{m_2}}{b}t_2\right) \quad (1)$$

where $\alpha_{m_1}, \alpha_{m_2}$ are the roots of $J_0(t_1)=0$ and $J_0(t_2)=0$ and m_1={1,2,3….. a}, m_2 = {1,2,3….. b}.
The coefficients $c(m_1,m_2)$ are given as,

$$c(m_1,m_2) = \frac{4\iint_{0\,0}^{a\,b} t_1 t_2 x(t_1,t_2) J_0\left(\frac{\alpha_{m_1}}{a}t_1\right) J_0\left(\frac{\alpha_{m_2}}{b}t_2\right) dt_1 dt_2}{(ab)^2[J_1(\alpha_{m_1})J_1(\alpha_{m_2})]^2} \quad (2)$$

where $J_1(\alpha_{m_1})$ and $J_1(\alpha_{m_2})$ are first order Bessel functions.

The Bessel coefficients $c(m_1,m_2)$ are unique for a given signal $x(t_1,t_2)$, similar to Fourier series coefficients [12]. The intervals between successive zero crossings of the Bessel function $J_0(t)$ increase slowly with time and approach the number π after sufficiently long time. However, unlike the sinusoidal basis functions in the Fourier series, the Bessel functions decay over time. This feature of the Bessel functions makes the Bessel series expansion suitable for analysis of non-stationary signals [13, 14, 15].

III. BESSEL TRANSFORM FOR IMAGE RESIZING

The method of resizing an image of size a **x** b to a size of $(a-m)$ **x** $(b-n)$ is said to be image down-sampling, where m and n are any positive integers and $m < a$ and $n < b$. Image up-sampling is the operation in which the size of the image is increased to a size of $(a+m)$ **x** $(b+n)$ [16]. In this section, we describe the formulation of both image down-sampling and image up-sampling.

A. Method for image down-sampling

Consider an image of size a **x** b,

(a) Calculate the Bessel coefficients of the image by using (2).

(b) The image is re-synthesized from Bessel coefficients in the interval $0 \leq t_1 \leq a - m,\ \ 0 \leq t_2 \leq b - n$ by using (3) to get the down-sampled version of the original image.

$$x_d\ (t_1, t_2) = \sum_{m_1=1}^{a} \sum_{m_2=1}^{b} c(m_1, m_2)\, J_0\left(\frac{\alpha_{m_1}}{a-m} t_1\right) J_0\left(\frac{\alpha_{m_2}}{b-n} t_2\right) \quad (3)$$

B. Method for image up-sampling

(a) Calculate the Bessel coefficients of the image by using (2).

(b) The image is re-synthesized from Bessel coefficients in the interval $0 \leq t_1 \leq a + m$, $0 \leq t_2 \leq b + n$ by using (4) to get the up-sampled version of the original image.

$$x_u(t_1, t_2) = \sum_{m_1=1}^{a} \sum_{m_2=1}^{b} c(m_1, m_2)\, J_0\left(\frac{\alpha_{m_1}}{a+m} t_1\right) J_0\left(\frac{\alpha_{m_2}}{b+n} t_2\right) \quad (4)$$

IV. IMAGE QUALITY EVALUATION

The image quality is evaluated in terms of MSE and PSNR, which are given as below for an 8-bit gray scale image.

$$MSE = \frac{1}{a_1 a_2} \sum_{i=1}^{a_1} \sum_{j=1}^{a_2} (x_{ij} - \hat{x}_{ij})^2 \quad (5)$$

$$PSNR = 10 log_{10}\left(\frac{255}{\text{MSE}}\right) \quad (6)$$

MSE represents the difference between original image and the processed image. Here $[x_{ij}]_{a_1 \mathbf{x}\, a_2}$ is the original image and $[\hat{x}_{ij}]_{a_1 \mathbf{x}\, a_2}$ is the reconstructed image and the size of the images is a_1 **x** a_2.

A. Performance evaluation for image up-sampling

A method for evaluating the performance of image up-sampling is given in [17]. We use this method for comparing the performance of different methods and the steps involved in it are given below:

(a) The original image I_{org} of size a **x** b is spatially down-sampled to get an image I_{dr} of size $a/2$ **x** $b/2$.

(b) Now the spatially down-sampled image I_{dr} is up-sampled by using the different methods which are to be compared, to obtain the images I_{us}.

(c) I_{org} acts as the reference to compare the images I_{us} generated using the different methods. MSE and PSNR values are computed between the images I_{org} and I_{us}.

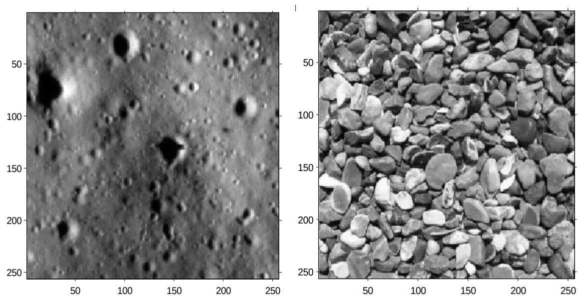
B. Performance evaluation for image down-sampling

A method for evaluating the performance of image down-sampling is given in [16, 17]. We use this method for comparing the performance of different methods and the steps involved in it are given below:

(a) The original image I_{org} of size a **x** b is spatially down-sampled to get an image I_{dr} of size $a/2$ **x** $b/2$.

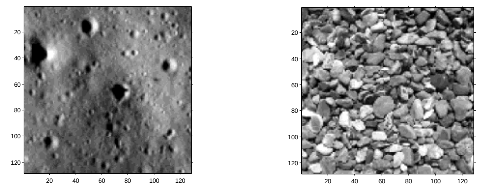
(b) Now the original image I_{org} is down-sampled by using the different methods which are to be compared, to obtain the images I_{ds}.

(c) I_{dr} acts as the reference to compare the images I_{ds} generated using the different methods. MSE and PSNR values are computed between the images I_{dr} and I_{ds}.



(a) (b)

Figure.1: Original images each of dimension 256 **x** 256 (a) Satellite and (b) Pebbles.



(a) (b)

Figure. 2: Spatially down sampled images each of dimension 128 **x** 128 (a) Satellite and (b) Pebbles.

V. RESULTS AND DISCUSSION

The fundamental difficulty in evaluating the performance of image resize technique is to decide which test images to be used for evaluation. We have performed the experiments on three different types of test images each of size 256 **x** 256, two of them are shown in Fig. 1. The spatially down-sampled images of size 128 **x** 128 are shown in Fig. 2. All the algorithms involved in the experiments are done using MAT-LAB. The proposed method is compared with the conventional methods like bilinear interpolation and bi-cubic interpolation.

Image resizing (up-scaling) operation is performed on spatially down-sampled images of size 128 **x** 128, through steps presented in Section IV.A to obtain the interpolated images of size 256 **x** 256. These interpolated images are shown in Fig. 3. The MSE and PSNR values of the resized (up-scaled) images generated by the different methods are given in Table 1. Image resizing (down-scaling) operation is performed on original test images of size 256 **x** 256, through steps presented in Section IV.B to obtain the decimated images of size 128 **x** 128. These decimated images are shown in Fig. 4. In Table 2 the MSE and PSNR values for image resizing (down-scaling) operation are given.

The images are exactly halved or doubled using the above mentioned methods is only to evaluate the performance of respective algorithms. In Fig. 5, images resized by different scale factors are shown. From Table 1, it can be inferred that the BT based image resizing gives better results when compared to bilinear interpolation and bi-cubic interpolation methods. The same can be inferred from Table 2 which is the case of image size reduction.

By observing the images shown in Fig. 3 (c), (f) and Fig. 4 (c), (f) it can be noted that BT based method preserves the perceptual quality of the image and also the edge information in the image. By looking at images shown in Fig. 5, we can perceptually make out that the bilinear interpolation and bi-cubic interpolation methods introduce blurring of sharp edges in the image, which is not the case in BT based method.

It should also be noted that there might be some mild artifacts in some parts of the images resized using BT based method. We can observe this in the 'pebbles' image shown in Fig. 3 (f) and there are no artifacts present in the 'satellite' image shown in Fig. 3 (c).

VI. SUMMARY AND CONCLUSIONS

The 2-D Bessel transform is rarely explored in image processing applications. We have presented an image resizing method based on Bessel transform and evaluated its performance. The proposed method yields a better result in terms of MSE, PSNR compared to the conventional resizing methods like bilinear interpolation and bi-cubic interpolation and also gives images of better perceptual quality and also does well in preserving the sharp edges of the image. It is also a simple and practical method to implement.

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Table 1: Comparisons of PSNR in dB and MSE results for various resizing methods of interpolation.

	Bilinear interpolation		**Bi-cubic interpolation**		**Bessel Transform**	
Images	***MSE***	***PSNR***	***MSE***	***PSNR***	***MSE***	***PSNR***
Satellite image	0.0013	52.76	0.0013	52.79	0.0008	55.12
Pebbels	0.0092	44.40	0.0095	44.20	0.0064	45.97
Lena	0.0018	51.54	0.0019	51.39	0.0011	53.51

Table 2: Comparisons of PSNR in dB and MSE results for various resizing methods of decimation.

	Bilinear interpolation		**Bi-cubic interpolation**		**Bessel Transform**	
Images	***MSE***	***PSNR***	***MSE***	***PSNR***	***MSE***	***PSNR***
Satellite image	0.0012	53.20	0.0011	53.80	0.000025	70.15
Pebbels	0.0085	44.75	0.0078	45.17	0.000015	72.03
Lena	0.0016	52.08	0.0015	52.34	0.000006	74.26

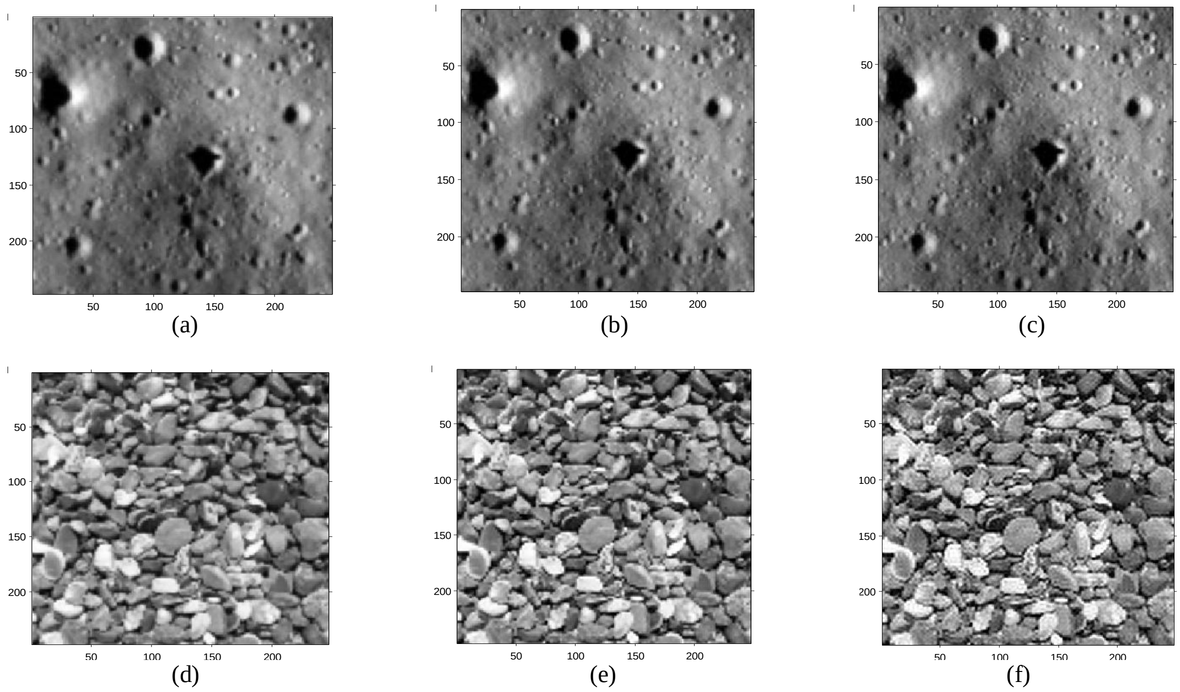

Figure. 3: Interpolated images (a), (d) using bilinear interpolation method, (b), (e) using bi-cubic interpolation method and (c), (f) using BT method.

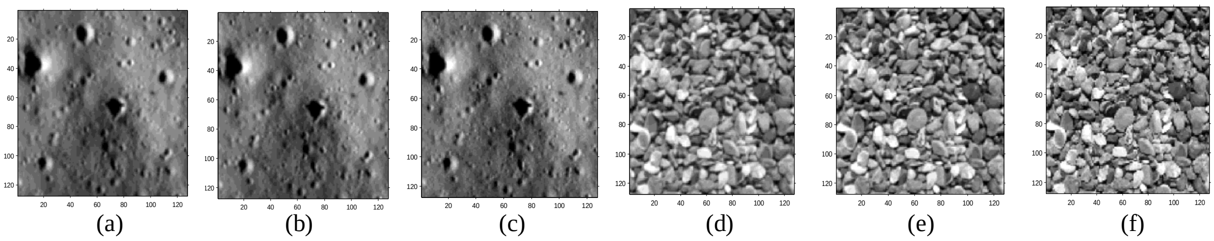

Figure. 4: Decimated images (a), (d) using bilinear interpolation method, (b), (e) using bi-cubic interpolation method and (c), (f) using BT method.

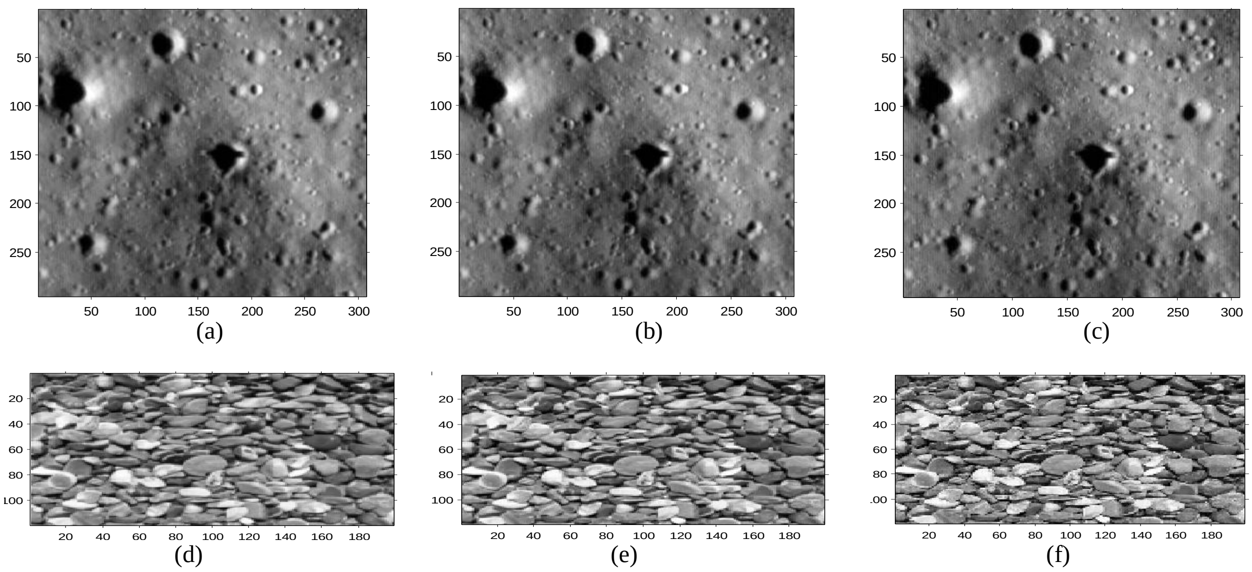

Figure. 5: Original images of size 256 x 256 resized to images of size (a) 292 x 308, (d) 130 x180 using bilinear interpolation method, (b) 292 x 308, (e) 130 x180 using bi-cubic interpolation method and (c) 292 x 308, (f) 130 x180 using BT method.