

NONITERATIVE TECHNIQUES FOR MINIMUM PHASE SIGNAL RECONSTRUCTION FROM PHASE OR MAGNITUDE

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New noniterative techniques are proposed for reconstruction of signal from samples of magnitude or phase of the Fourier transform of the signal. The only condition for reconstruction is that the signal is a minimum phase one. The basis for these new techniques is the relation between the magnitude and phase functions through cepstral coefficients. The techniques are illustrated through several examples. In all the cases we find that phase from magnitude can be obtained exactly and magnitude from phase can be obtained to within a scale factor. Effects of truncation of minimum phase signals and aliasing due to sampling in the frequency domain are discussed. These studies show that effective noniterative techniques can be evolved for signal reconstruction instead of cumbersome iterative procedures suggested in literature recently.

I. INTRODUCTION

Recently iterative techniques are proposed for signal reconstruction from magnitude or phase of the Fourier transform of a minimum phase signal [1]. The objective of this paper is to show that the signal reconstruction is possible using noniterative procedures. The same procedures can also be used for the computation of Hilbert transform of log-magnitude or phase of the Fourier transform of a minimum phase signal. The noniterative procedures are based on two key ideas. The first idea is to use the relation between log magnitude and phase functions through cepstral coefficients [2]. The second one is a new phase unwrapping procedure discussed in this paper.

Iterative techniques for minimum phase signal reconstruction proposed in [1] involve repeatedly imposing the causality constraint in the time domain and incorporating the known phase or magnitude in the frequency domain. The iterative algorithm for signal reconstruction from phase results in a minimum phase sequence that is accurate to within a scale factor, whereas the iterative algorithm for signal reconstruction from magnitude yields a unique minimum phase sequence. The convergence

of these two algorithms has not been firmly established, although they seem to converge for the examples considered by the authors in [1].

In this paper we propose noniterative techniques which accomplish the task of minimum phase signal reconstruction from magnitude or phase of the Fourier transform of the signal. The minimum phase condition is imposed so that the magnitude (or phase) function can be derived from the given phase (or magnitude) function. The conditions of causality and/or finite duration are not explicitly imposed. The signal reconstruction from the magnitude function is unique. The reconstructed signal from the given phase is determined to within a scale factor. The conditions on signals given in [1] are applicable for our reconstruction algorithms also.

The techniques proposed in this paper are based on the relations between log magnitude and phase of a minimum phase signal through cepstral coefficients. These relations are discussed in [3]. A new phase unwrapping method is proposed here which is based on the relations given in [4]. The method is somewhat similar to the one given in [5], except that our method is nonadaptive and noniterative. Our signal reconstruction results are valid only for minimum phase signals, whose properties are discussed in [1] and, therefore, they will not be repeated here.

In section II we discuss the problem of minimum phase signal reconstruction and the basis for our noniterative techniques. The techniques are illustrated through examples in section III.

II. BASIS FOR THE NONITERATIVE TECHNIQUES

Problem of Minimum Phase Signal Reconstruction

Let $V(\omega)$ be the Fourier transform of a minimum phase sequence $v(n)$ of length N samples. That is

$$V(\omega) = \sum_{n=0}^{N-1} v(n) e^{-j\omega n} \quad (1)$$

Also, let $V(\omega) = |V(\omega)| e^{j\theta_V(\omega)}$

Note that, $V(\omega)$ is periodic in ω with period 2π . The problem of our interest in signal reconstruction can be stated as follows:

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Given the magnitude function $|V(\omega)|$ or the phase function $\theta_V(\omega)$, how to obtain the sequence $v(n)$.

To reconstruct the sequence $v(n)$, we require both the magnitude $|V(\omega)|$ and the phase $\theta_V(\omega)$. Given $|V(\omega)|$ alone or $\theta_V(\omega)$ alone, the first step in signal reconstruction is to obtain $\theta_V(\omega)$ from $|V(\omega)|$ or $|V(\omega)|$ from $\theta_V(\omega)$. It is possible to accomplish this first step using Hilbert transform relations given in [6]. But computation of Hilbert transform is quite involved and moreover, to recover $|V(\omega)|$ from $\theta_V(\omega)$, we require the unwrapped phase. Normally, the phase function available in these problems is the principal value of the phase only, i.e., all the phase values lie in the range $-\pi$ to π .

An alternative to Hilbert transform is the use of the relation between $\log |V(\omega)|$ and $\theta_V(\omega)$ through cepstral coefficients, which is described in [7] and will be briefly discussed here.

Relation between Logmagnitude and Phase through Cepstral Coefficients

For a minimum phase signal $\ln V(\omega)$ can be written as

$$\ln V(\omega) = c(0)/2 + \sum_{n=1}^{\infty} c(n) e^{-j\omega n} \quad (3)$$

where $\{c(n)\}$ are called cepstral coefficients. Using (2) and (3) we get

$$\ln |V(\omega)| = c(0)/2 + \sum_{n=1}^{\infty} c(n) \cos \omega n \quad (4)$$

and

$$\theta_V(\omega) = \theta(\omega) + 2\lambda\pi = -\sum_{n=1}^{\infty} c(n) \sin \omega n, \quad (5)$$

where λ is an integer.

Taking the negative derivative of $\theta_V(\omega)$ with respect to ω we obtain the group delay function $\tau(\omega)$.

$$\tau(\omega) = -d\theta_V(\omega)/d\omega = \sum_{n=1}^{\infty} n \cos \omega n \quad (6)$$

Equations (4) and (5) show that the log magnitude and the phase of a minimum phase sequence are related through cepstral coefficients. These relations together with (6) form the basis for our noniterative algorithms for signal reconstruction.

Briefly, the procedure for signal reconstruction is as follows. Given $|V(\omega)|$, compute $\{c(n)\}$ using (4) and then use the $\{c(n)\}$ to obtain $\theta_V(\omega)$ from (5). If $\theta_V(\omega)$ is given, then compute $\{c(n)\}$ using (5) and then use the $\{c(n)\}$ to compute $|V(\omega)|$ from (4). Computation of $\{c(n)\}$ from $\theta_V(\omega)$ using (5) requires the phase function to be given in unwrapped form. Usually, only the principal value phase function $\theta(\omega)$ will be available. The true phase function $\theta_V(\omega)$ is obtained from $\theta(\omega)$ through a procedure called phase unwrapping. The procedure involves computation of the all-pass sequence $x(n)$ from the given samples of $\theta(\omega)$. The phase derivative is computed using the discrete Fourier transforms (DFT) of $\{x(n)\}$

and $\{nx(n)\}$ as described in [5]. From the phase derivative, the cepstral coefficients are obtained using the relation (6). From the cepstral coefficients, samples of the unwrapped phase are obtained through DFT. Note that the constant and linear components of phase are not included in the resulting unwrapped phase values. The phase unwrapping and signal reconstruction procedures can be implemented through N-point DFTs.

iii. EXAMPLES

In this section we illustrate the signal reconstruction techniques through several examples.

Example 1 : Unit sample Response of an All-pole System

We consider the unit sample response of a sixth order all-pole digital filter corresponding to three resonators in cascade. We assume a sampling frequency of 10kHz. Only the first 256 samples of the response are used for illustration. Beyond 256 samples the response has decayed to effectively zero and hence it is ignored. Appending the signal with 256 zeros, a 512 point sequence is created. The samples of magnitude and phase of the Fourier transform are obtained through a 512-point DFT.

The samples of magnitude are used to reconstruct the signal. In this case the derived phase and the reconstructed signal are exact. When the samples of phase are used to reconstruct the signal the derived magnitude and the reconstructed signal are again exact. Noniterative techniques proposed in this paper seem to work very well for this example. This example is similar to the example discussed in [1] to illustrate iterative algorithms for minimum phase signal reconstruction.

Example 2 : Unit sample Response of an All-pole System : Effect of Truncation

There are two important consequences of DFT realization of the reconstruction algorithms : (1) only a finite number of samples of the signal can be reconstructed; (2) The given input data consists of samples of magnitude or phase functions in the frequency domain. For a system with poles, the unit sample response is infinite. Signal reconstruction using the DFT samples of a truncated signal will not be exact. The effect of truncation is illustrated by considering the first 64 samples of the unit sample response in Example 1. A 128-point FFT algorithm is used for this illustration. Signal reconstructed from magnitude samples is close to the original, whereas the signal reconstructed from phase samples is not exact. The errors due to truncation are more severe in the signal reconstructed from phase than from magnitude.

Example 3 : Unit sample Response of an All-zero System : Effect of Aliasing

We consider the unit sample response of an all-zero system consisting of three complex conjugate pairs of zeros, all within the unit cir-

cle in the z -plane. This example is chosen to study the effects of the order of DFT used in the reconstruction algorithms. Since the unit sample response is finite, there is no effect of truncation as in Example 2. The samples of phase and magnitude of the Fourier transform of the given sequence are obtained for different values of N , by appending the sequence appropriately with zeros. The original and the sequence reconstructed are given in Table I. The accuracy of the reconstructed sequence is measured by computing the mean-squared error between the reconstructed sequence and the original sequence for the first seven samples. For signals reconstructed from phase, the reconstructed sequences is multiplied with an optimum scale factor α , which gives the least error. The error progressively increases as the number points (N) used for DFT computation is reduced. This is to be expected because the aliasing in the computed cepstrum will increase with decreasing N [6]. The aliasing error will be more in the case of signal reconstruction from samples of phase than in the case of reconstruction from magnitude. This is because of the computation of the sequence $\{c(n)\}$ using DFT of the samples of phase derivative. The results in Table I show that the error due to aliasing is low even for $N=16$ for the specific sequence considered in this example. It is possible that, for signals having sharp changes in the phase derivative function, the aliasing effect may be more severe.

Results of signal reconstruction using the iterative algorithms described in [1] are also given Table I for comparison with the results of our noniterative techniques. The error in the signal reconstructed even after 10 iterations is larger than the error in the signal reconstructed by our noniterative techniques. The error can be reduced using a combination of noniterative and iterative methods. Samples of the unknown phase (or magnitude) are first obtained by the noniterative techniques. These samples are used to begin iteration in the iterative algorithms instead of using unity magnitude (or zero phase). This method yields a significant reduction in error as shown in the Table I for the sequence derived from samples of phase using $N=16$.

IV. SUMMARY AND CONCLUSIONS

In this paper we described noniterative techniques for signal reconstruction from samples of magnitude or phase of the Fourier transform of a minimum phase signal. These techniques do not explicitly impose constraints of causality or finite duration on the signal as in the iterative techniques [1]. The disadvantage of the iterative algorithms, namely, convergence problems and large number of iterations, are not present in our techniques. From the samples of the magnitude, the cepstral coefficients can always be computed. So in this case there may not be any need for iterative procedures as proposed in [1]. The reconstruction of signal from phase, however, involves determination of unwrapped phase first. The phase unwrapping

procedure proposed in this paper can be used for this purpose.

The principles in developing the signal reconstruction algorithms can also be used for obtaining the Hilbert transform of phase or log magnitude. In fact, while deriving phase from magnitude and vice versa, we have obtained the Hilbert transforms as intermediate results in our algorithms.

The techniques suggested in this paper are applicable only for minimum or maximum phase signals. Generally most signals are mixed phase either due to truncation of an infinite duration unit sample response or due to some of the system poles or zeros being outside the unit circle in the z -plane. For mixed phase signals the magnitude and phase are not related through cepstral coefficients as given in this paper. In such cases the reconstructed signal will be only the minimum phase equivalent. However, our phase unwrapping procedure is applicable even for mixed phase signals also.

We find that it is possible to extend the concepts presented in this paper for mixed phase signal reconstruction also. Using the ideas discussed in the context of pole-zero decomposition in [7], we feel that it is possible to establish a relation between the magnitude and phase of the Fourier transform of certain categories of mixed phase signals. This relation may be useful to develop either noniterative or fast converging iterative algorithms for mixed phase signal reconstruction. We are currently exploring some of these possibilities.

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TABLE I : Signal reconstruction from magnitude and from phase of the Fourier transform of a finite duration signal (unit sample response of an all-zero system) : Effect of aliasing

	X(0)	X(1)	X(2)	X(3)	X(4)	X(5)	X(6)	Mean Squared Error
Original signal	1.0000	-0.6080	1.5387	-0.3556	1.4772	-0.5454	0.8681	
From magnitude	1.0000	-0.6080	1.5387	-0.3556	1.4772	-0.5454	0.8681	0.4×10^{-8}
From phase	0.9998	-0.6077	1.5384	-0.3552	1.4769	-0.5451	0.8679	1.4×10^{-7}
From magnitude	0.9995	-0.6079	1.5377	-0.3556	1.4778	-0.5455	0.8692	0.9×10^{-4}
From phase	1.0000	-0.5397	1.5471	-0.2352	1.4837	-0.4779	0.8603	3.0×10^{-3}
From magnitude	1.1164	-0.6833	1.6196	-0.3120	1.3127	-0.4707	0.8225	0.9×10^{-2}
From phase	0.8951	-0.6712	1.6123	-0.3624	1.5750	-0.5198	0.6196	1.3×10^{-2}
From phase; result after 10 iterations starting with unity magnitude	1.0900	-0.3480	1.5058	-0.2527	1.4465	-0.3806	0.9702	2.0×10^{-2}
From phase; result after 10 iterations starting with magnitude derived from phase noniteratively.	0.8950	-0.5498	1.5633	-0.4212	1.4868	-0.4885	0.7579	0.5×10^{-2}

Signal reconstruction using

512-pt FFT
128-pt FFT
16-pt FFT