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Analytic phase-based representation for face recognition

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***Abstract*—A representation based on the phase of analytic image is proposed to address the issue of illumination variation in face recognition task. The problem of unwrapping in the computation of analytic phase is avoided by using trigonometric functions of phase. Template matching is used to compare the functions of analytic phase for face recognition. For template matching, the functions of the analytic phase are compressed using eigenanalysis. Performance of the face recognition is improved by using weights derived from the eigenvalues in the template matching.**

I. Introduction

The objective of face recognition task is to recognize a person using his/her face image information [1]. The issues involved in this task are inter-class variation and intra-class variation. Inter-class variation refers to the difference between face images of two persons, which is due to unique information present in the human face image. Intra-class variation refers to the differences in the face images of a given person under varying conditions of pose, illumination and expressions [1]. In this paper we address the issue of variation due to illumination in face recognition.

Changes in illumination are normally dealt with either by modeling the effect of illumination of faces, or by extracting features that are less sensitive to illumination. Modeling the effect of illumination requires samples of the face image for various illumination conditions. These approaches are discussed in [2], [3]. Another drawback of these approaches is that they may introduce artifacts that may smear some unique information of the person's face. These artifacts may in turn decrease the performance of face recognition. On the other hand, extracting features that are less sensitive to illumination requires suitable representation of the face image. Some of the representations are given in [4], [5]. All these representations are based on the edge map of the given face image.

Edge information can also be represented by the phase of the analytic image. But the issue of unwrapping the phase of analytic image needs to be addressed before using it. Phase unwrapping is avoided by using trigonometric functions of phase of the analytic image. These functions cannot be used directly for template matching because they give poor performance under the intra-class variation. Eigenanalysis is applied on the functions of analytic phase to derive a suitable representation of these functions for template matching to improve the performance of the face recognition.

Performance of the proposed approach is evaluated using illumination angle variation of the FacePix database [6], [7]. The illumination set in this database was captured with the subject looking directly into the camera, while the light source was moved around the subject. The light source was moved at 1° interval from -90° to 90°. These images are denoted by $L^1, \ldots, L^{181}$. The sizes of the face images are rescaled to 50×50 pixels in our experiments.

The proposed representation of the face image is described in Section II. Eigenanalysis on the proposed representation is discussed in Section III. Section IV gives the results of experimental studies. A summary of the work is given in Section V.

II. Analytic image

The analytic signal of a real one-dimensional (1-D) signal was proposed by Gabor in 1946 [8]. This idea was extended to two dimensional signals and was applied for texture classification and segmentation [9]. There are three definitions proposed toward this aim [10]. One of the definitions of analytic signal in two-dimension is as follows:

Let $x(t_1, t_2)$ is the given face image, then the analytic image can be written as [10]

$$\begin{aligned} x_A(t_1,t_2) &= x(t_1,t_2) + j\mathcal{H}\{x(t_1,t_2)\}, \\ &= x(t_1,t_2) + jx_H(t_1,t_2), \end{aligned} \quad (1)$$

where

$$\begin{aligned} x_H(t_1,t_2) &= \mathcal{H}\{x(t_1,t_2)\} \\ &= \text{p.v}\int\int \frac{1}{\pi^2}\frac{x(\tau_1,\tau_2)}{(t_1-\tau_1)(t_2-\tau_2)}d\tau_1 d\tau_2, \end{aligned}$$

is the Hilbert transform of the image $x(t_1, t_2)$. This definition does not satisfy exactly the properties of 1-D analytic signal [10]. The definition of analytic signal for 2-D signal is still a research issue [10]. We have followed the definition given by (1) in this work.

Equation (1) can be broken into two components, namely magnitude and phase as

$$\begin{aligned} x_A(t_1,t_2) &= x(t_1,t_2) + jx_H(t_1,t_2) \\ &= |x_A(t_1,t_2)|\exp(\phi(t_1,t_2)), \end{aligned} \quad (2)$$

where $|x_A(t_1,t_2)| = \sqrt{x(t_1,t_2)^2 + x_H(t_1,t_2)^2}$, and $\phi = \arctan\{\frac{x_H(t_1,t_2)}{x(t_1,t_2)}\}$. It is difficult to visualize how the information in these two components are related, because the

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magnitude and phase are not directly comparable. Analytic phase ($\phi(t_1, t_2)$) is used to characterize the sharp variations in texture and fingerprint images [9]. The computation of analytic phase using $\arctan$ gives the wrapped phase, as this computation gives only the principal value in the range $(-\pi, \pi]$. Any value outside this interval will be wrapped around to produce the wrapped phase ($\phi_w(t_1, t_2)$), which is related to actual phase $\phi(t_1, t_2)$ by

$$\phi(t_1, t_2) = \phi_w(t_1, t_2) \pm l(t_1, t_2) 2\pi, \quad (3)$$

for some integer $l(t_1, t_2)$. This issue was addressed in the literature using phase unwrapping and group-delay processing [11], [12]. In this work, the phase unwrapping is avoided using trigonometric functions of phase computed as

$$x^s(t_1, t_2) = \sin(\phi(t_1, t_2)) = \frac{x_H(t_1, t_2)}{\sqrt{x^2(t_1, t_2) + x_H^2(t_1, t_2)}},$$

and

$$x^c(t_1, t_2) = \cos(\phi(t_1, t_2)) = \frac{x(t_1, t_2)}{\sqrt{x^2(t_1, t_2) + x_H^2(t_1, t_2)}}. \quad (4)$$

These two functions of the analytic phase of a face image are shown in Fig. 1. It shows that the two functions contain complementary information. Thus they are used separately for template matching. The smearing of edges in the functions of analytic phase while using eigenanalysis is explained in next section.

III. Eigenanalysis using functions of phase of analytic image

Let the training face images of the i^{th} person be denoted by the set D_i. For all the available training face images the $\sin$ and $\cos$ functions of the analytic phase are computed using (4). Let $\mathbf{x}^c$ and $\mathbf{x}^s$ be vector representations of the $\cos$ and $\sin$ functions of the analytic phase respectively, for the given face image $x(t_1, t_2)$. The eigenvector matrix $\mathbf{\Psi}^s \in \mathbb{R}^{N \times N} = [\psi_1^s, \dots, \psi_N^s]$, and diagonal eigenvalue matrix $\mathbf{\Lambda}^s \in \mathbb{R}^{N \times N} = diag\{\lambda_1^s, \lambda_2^s, \dots, \lambda_N^s\}$ with diagonal elements (eigenvalues) in decreasing order ($\lambda_1^s \geq \lambda_2^s \dots \geq \lambda_N^s$), are computed using $\mathbf{x}^s$ representations of the given training face images. Similarly $\mathbf{\Psi}^c \in \mathbb{R}^{N \times N}$ and $\mathbf{\Lambda}^c \in \mathbb{R}^{N \times N}$ are computed using $\mathbf{x}^c$ representations of face images. Here $N = R \times C$, where *R* and *C* are the number of rows and columns of a given face image, respectively. The eigenvectors are used to represent the given face image $x(t_1, t_2)$ as follows:

$$\mathbf{a}_{\mathbf{x}}^{s,N} = (\mathbf{\Psi}^s)^t \mathbf{x}^s$$
$$\mathbf{a}_{\mathbf{x}}^{c,N} = (\mathbf{\Psi}^c)^t \mathbf{x}^c. \quad (5)$$

These new representations also known as projected coefficients, are used for template matching for face recognition task. If we use all the coefficients (all *N* values of the vectors $\mathbf{a}_{\mathbf{x}}^{s,N}$ and $\mathbf{a}_{\mathbf{x}}^{c,N}$), the matching of face images will be poor, as this will not address the issue of intra-class variation. The matching can be improved by slightly smearing the edges while matching. Smearing of the edges takes place when only the first few coefficients are considered. But removing many coefficients may lead to loss in the identity information of that face image. Thus there is a trade-off in the choice of the number of coefficients in eigenanalysis.

The Euclidean distance measure is used in template matching. Let $d_{i,\mathbf{y}}^{c,m}$ denote the minimum Euclidean distance obtained for a given test face image $y(t_1, t_2)$ using $\cos$ function of the analytic phase (first *m* coefficients) of the available training face images of the i^{th} person. It can be written as

$$d_{i,\mathbf{y}}^{c,m} = \min_{\mathbf{x} \in D_i} ||\mathbf{a}_{\mathbf{y}}^{c,m} - \mathbf{a}_{\mathbf{x}}^{c,m}||_2. \quad (6)$$

Similarly, the minimum Euclidean distance is computed using $\sin$ function of the analytic phase of the test and training face images, and is denoted by $d_{i,\mathbf{y}}^{s,m}$. The identity (i^*) of a given face image is obtained using the combined Euclidean distance as follows:

$$i^* = \arg\min_i [(d_{i,\mathbf{y}}^{s,m})^2 + (d_{i,\mathbf{y}}^{c,m})^2]^{1/2}. \quad (7)$$

The performance (η) of face recognition is computed as

$$\eta = \frac{\text{Number of correctly identified face images}}{\text{Total number of available face images}} \times 100. \quad (8)$$

One can reconstruct the $\cos$ function of the analytic phase of a given face image using the projected coefficients ($\mathbf{a}_{\mathbf{x}}^{c,N}$) as follows

$$\mathbf{x}^c = \mathbf{\Psi}^c \mathbf{a}_{\mathbf{x}}^{c,N} = \sum_{i=1}^{N} a_{\mathbf{x},i}^{c,N} \psi_i^c, \quad (9)$$

where $\mathbf{a}_{\mathbf{x}}^{c,N} = [a_{\mathbf{x},1}^{c,N}, \dots, a_{\mathbf{x},N}^{c,N}]$. This representation can be divided into three terms as

$$\mathbf{x}^c = \sum_{i=1}^{l_1} a_{\mathbf{x},i}^{c,N} \psi_i^c + \sum_{i=l_1+1}^{l_2} a_{\mathbf{x},i}^{c,N} \psi_i^c + \sum_{i=l_2+1}^{N} a_{\mathbf{x},i}^{c,N} \psi_i^c, \quad (10)$$

where l_1 and l_2 are indices. The first term defined by the coefficients $\{a_{\mathbf{x},i}^{c,N}, \quad i = 1, \dots, l_1\}$, corresponds to the information which is common to all the training face images ($\cos$ function of the analytic phase). The second term corresponds to the unique information present in that face image ($\cos$ function of the analytic phase) and defined by the coefficients $\{a_{\mathbf{x},i}^{c,N}, \ i = l_1+1, \dots, l_2\}$. The third term contains mostly noise only. Thus for discrimination, the coefficients corresponding to the second term will be more useful as compared to the coefficients of first and third terms. The third and first terms do not have any discriminative information of the face image. It can be justified by reconstructing the $\cos$ of the analytic phase for different number of coefficients. Fig. 2 shows the approximately reconstructed $\cos$ of analytic phase $\hat{\mathbf{x}}^c = \sum_{i=l_1}^{l_2} a_{\mathbf{x},i}^{c,N} \psi_i^c$ for different values of l_1 and l_2. One can observe from the figures that it is easy to discriminate between face images (Fig. 2 (a) and (d)) using representations shown in Fig. 2 (c) and (f) as compared to Fig. 2 (b) and (e). Based on these observations, one can improve the performance of the face recognition by assigning more weights to the coefficients in the second term as compared to the coefficients of first and third terms for matching. One of the issues that need to be

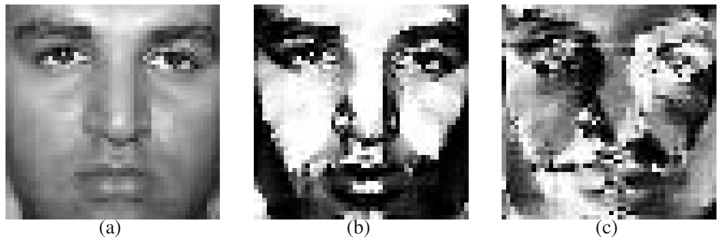

Fig. 1. (a) Gray level image. (b) Cos of analytic phase, and (c) Sine of analytic phase.

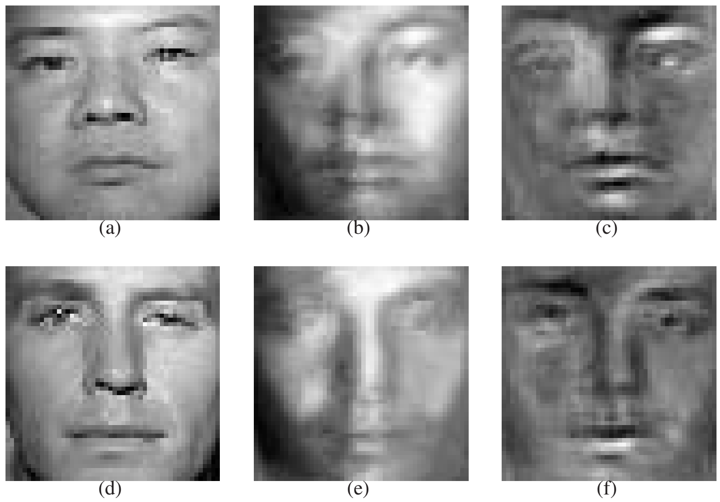

Fig. 2. Illustration of the importance of eigenvectors. (a) Gray level face image. Reconstruction of cos of analytic phase of face image using eigenvectors: (b) 1 to 5, and (c) 6 to 10. (d) Gray level face image. Reconstruction of cos of analytic phase of face image using eigenvectors: (g) 1 to 5, and (h) 6 to 12.

addressed is how to decide the values of the indices l_1 and l_2 to divide the given face image representation in three terms as discussed above. The indices l_1 and l_2 are generally specific for a given face image.

In this work the eigenvalues are used to decide the weights to the projected coefficients in template matching. In general the eigenvalues decay rapidly initially. The inverse of the eigenvalues are used as the weights to the projected coefficients in matching. This will give more weight to the coefficients which have more discriminative information of a given face image. Following these ideas the weighted Euclidean distance will be

$$\begin{aligned} d_{i,\mathbf{y}}^{s,m} &= \min_{\mathbf{x}\in D_i} ||\mathbf{a}_{\mathbf{y}}^{s,m}\mathbf{W}_m^s - \mathbf{a}_{\mathbf{x}}^{s,m}\mathbf{W}_m^s||_2 \\ d_{i,\mathbf{y}}^{c,m} &= \min_{\mathbf{x}\in D_i} ||\mathbf{a}_{\mathbf{y}}^{c,m}\mathbf{W}_m^c - \mathbf{a}_{\mathbf{x}}^{c,m}\mathbf{W}_m^c||_2, \end{aligned} \tag{11}$$

where $\mathbf{W}_m^c = diag\{\frac{1}{\sqrt{\lambda_1^c}}, \ldots, \frac{1}{\sqrt{\lambda_m^c}}\}$, and $\mathbf{W}_m^s = diag\{\frac{1}{\sqrt{\lambda_1^s}}, \ldots, \frac{1}{\sqrt{\lambda_m^s}}\}$ are diagonal matrices. One has to take care of the weights obtained with eigenvalues close to zero. In this work we have assigned a very small positive value for those eigenvalues.

IV. Experiments

The performance of the $\sin$ and $\cos$ functions of the analytic phase using the two distance measures for different sets of training face images of the FacePix database is shown in Table I. The performance using weighted Euclidean distance is better than normal Euclidean distance for both the functions of the analytic phase. The $\sin$ function of the analytic phase gives better performance than the $\cos$ function of the analytic phase. The reason could be that the $\cos$ function is similar to original gray level face image as compared to the $\sin$ function. Another observation that one can make from the table is that combining the $\sin$ and $\cos$ functions of the analytic phase does not improve the performance in case of one training face image. For comparison, the performances with existing approaches [6], [7] are also given in the same table. It shows that the performance of the proposed approach is better than that of the existing approaches, except for the case of one training face image.

V. Summary

The issue of illumination variation in face recognition task is addressed using trigonometric functions ($\sin$ and $\cos$) of analytic phase. These functions are analyzed separately

TABLE I

RECOGNITION RATE (IN %) ON FACEPIX DATABASE. HERE E = EUCLIDEAN DISTANCE, WE = WEIGHTED EUCLIDEAN DISTANCE. THE COMPUTATION OF THE PERFORMANCES WITH PCA, LDA, HMM, AND BIC ARE GIVEN IN [6], [7]

Representation	WE/E	Set of training face images		
		L^{91}	L^1, L^{91} and L^{181}	L^1, L^{46}, L^{91},L^{136}, and L^{181}
Cos	E	33.84	70.6	93.7
	WE	39.2	92.7	99.13
Sine	E	60.75	86.84	99.04
	WE	61.93	95.65	99.27
Combine	E	48.5	85.3	98.28
	WE	51.5	96.67	99.8
PCA		48.84	71.71	90.33
LDA		53.04	79.52	94.92
HMM		19.26	37.38	59.37
BIC		49.80	79.10	93.54
Edginess [5]		81.53	94.32	99.72

because they contain some complementary information of the given face image. The proposed approach avoids the problem of unwrapping in the computation of analytic phase. These functions are used in template matching based approach for face recognition. The template matching-based approach give poor performance under the intra-class variation. It is improved by smearing the edges in the images functions of the analytic phase. The smearing of edges is performed by considering only the first few coefficients obtained from eigenanalysis. The performance is further improved by assigning weights to the projected coefficients in the template matching. The experimental results show that the proposed representation exploits the analytic phase effectively, and provide results comparable to the existing approaches.

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