Signal Processing 19 (1990) 139-149
Elsevier

SIGNAL RECONSTRUCTION FROM PARTIAL DATA FOR SENSOR ARRAY IMAGING APPLICATIONS

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Received 16 May 1988
Revised 20 February 1989 and 11 September 1989

Abstract. In this paper, the problem of signal recovery from partial information with special reference to sensor array imaging situations is examined. In sensor array imaging, as used for viewing underwater objects, the complex wave field is measured at a discrete set of points on a receiver airier. This data is used to reconstruct an image of the object using a suitable transformation. This data can be viewed as partial magnitude and phase data, which after appropriate phase modification, is transformed to obtain the object information. Recently, algorithms have been proposed in the literature for reconstruction of signals from partial information such as magnitude or phase (1-bit phase in some cases) of the Fourier transform. We examine the relevance of these algorithms to sensor array imaging situations. We propose new algorithms that are applicable to these situations. In particular, we show that measurement at multiple frequencies with suitable phase quantization reduces the receiver complexity and sometimes the effects of noise. The proposed new algorithms are demonstrated with several illustrations of simulated field data for a simplified model of an imaging setup.

Zusammenfassung. In diesem Paper untersuchen wir das Problem der Signalrückgewinnung von Teilinformationen mit speziellem Bezug zur akustischen Abbildungssituation. Bei akustischer Abbildung, wie zur Betrachtung von Unterwasserobjekten, wird das komplexe akustische Feld an diskreten Punkten eines Empfangsarrays gemessen. Diese Daten werden zur Rekonstruktion des Objekts mittels einer geeigneten Transformation verwendet. Dies ergibt, daß diese Daten als Teilamplituden- und -phasendaten angegeben werden können, die nach geeigneter Phasenmodifikation transformiert werden, um die Objektinformation zu erhalten. Vor kurzem wurden in der Literatur Algorithmen zur Signalrekonstruktion aus Teilinformation wie Amplitude oder Phase (1 bit Phase in einigen Fällen) der Fouriertransformation vorgeschlagen. Wir untersuchen die Bedeutung dieser Algorithmen für die Situation der akustischen Abbildung. Wir schlagen neue Algorithmen vor, die für diese Gegebenheiten anwendbar sind. Im besonderen zeigen wir, daß Messungen bei mehreren Frequenzen und geeigneten Phasenquantisierungen die Komplexität des Empfängers und die Rauscheinflüsse verminderen. Die vorgeschlagenen neuen Algorithmen werden mit verschiedenen Illustrationen von simulierten Schallfelddaten dargestellt.

Résumé. Nous examinons dans cet article le problème du recouvrement d'un signal à partir d'une information partielle dans le cas particulier de la reconstruction visuelle à partir d'un réseau de capteurs (en anglais: sensor array imaging). Dans ce contexte, à des fins de visualisation d'objets sousmarins, un champ d'ondes complexe est mesuré sur un ensemble discret de points constituant un réseau de réception. Ces données sont utilisées pour reconstruire l'image de l'objet à l'aide d'une transformation adéquate. Ces données peuvent être considérées comme des données partielles d'amplitude et de phase, qui, après une modification appropriée de cette dernière, sont transformées pour obtenir l'information sur l'objet. Récemment, des algorithmes de reconstruction de signaux à partir d'une information partielle telle que l'amplitude ou la phase de la transformée de Fourier (phase codée sur un bit dans certains cas) ont été proposés dans la littérature spécialisée. Nous examinons la pertinence de ces algorithmes dans le contexte de la reconstruction visuelle à partir d'un réseau de capteurs. Nous proposons de nouveaux algorithmes applicables à ce problème. En particulier, nous montrons qu'une mesure à des fréquences multiples associée à une quantification de la phase bien choisie permet de réduire la complexité du récepteur et quelquefois les effets du bruit. Les possibilités des nouveaux algorithmes proposés sont illustrées sur plusieurs champs simulés avec un modèle simplifié de dispositif de reconstruction visuelle.

Key words: Sensor array imaging, projection onto convex sets (POCS), phase quantisation, 1-bit phase, 2-bit phase, multiple frequency data.

1. Introduction

In this paper we examine the problem or image reconstruction with special reference to some signal processing issues that arise in the context of sensor array imaging. We consider a simplified model of the imaging problem to focus our attention on signal processing issues in imaging. The major problem in sensor array imaging is to reconstruct an image from sparse data which are complex and noisy. The objective is to develop algorithms to overcome the effects of sparse partial data in order to get an image of acceptable quality for recognition of objects in the image. In imaging, the complex field data are multiplied by a phase factor corresponding to the propagation effect before transforming the data to obtain an image. On the other hand, in image processing, manipulations are done on the image intensity values available in the form of pixel data. Therefore imaging and image processing are different, although both may use operations such as two-dimensional Fourier transformation. Standard image processing techniques like noise cleaning and edge enhancement cannot be used on images obtained in sensor array imaging because of the poor resolution due to sparse data.

The complex noisy field data measured at discrete points on a receiver array can be viewed as partial magnitude and phase data. We propose new algorithms for image reconstruction from these partial data and show that the quality of the reconstructed image can be significantly improved using these algorithms. We also suggest methods to reduce the receiver complexity significantly, by using a small number of sensor elements. Measurements at multiple frequencies and quantization of phase data reduces the receiver complexity and the effects of noise in the data. We demonstrate these results through simulation studies.

The image formation in sensor array imaging is briefly described as follows: The signal g, arriving at a point (x, y) in the Fresnel zone, due to an object in the plane (x_0, y_0) is given by [10]

$$g(x, y) = \iint_{-\infty}^{\infty} g_0(x_0, y_0) \exp(jkr)\, dx_0\, dy_0, \tag{1}$$

where k is the wavenumber and r is the distance vector between the points on the object and the receiver planes. It can be shown that this equation can be written as a convolution equation in the following manner:

$$g(x, y) = g_0(x, y) * h(x, y). \tag{2}$$

Convolutions are easily handled by Fourier transforms. In our case it so because we know one of the factors completely. Taking the Fourier transform of both sides of (2), we get

$$G(f_1, f_2) = G_0(f_1, f_2) H(f_1, f_2). \tag{3}$$

The factor $H(f_1, f_2)$ is given by

$$H(f_1, f_2) = \exp[jkz\{1-(\lambda f_1)^2-(\lambda f_2)^2\}^{1/2}], \tag{4}$$

where λ is the wavelength of the incident wave used for imaging. Therefore, to recover the original signal from the received data, the following procedure is adopted:

(1) Take the Fourier transform of $g(x, y)$.

(2) Multiply this by $H^{-1}(f_1, f_2)$.

(3) Take the inverse Fourier transform of the resultant.

The above procedure for image formation is called the backward propagation technique [10]. This is the theoretical basis for our simulation studies. In order to focus on the implications of sparse data, we consider only a simplified model of the imaging problem ignoring several practical issues. We restrict our attention to imaging by the holographic technique, which enables the use of sophisticated signal processing. Figure 1 shows a model of a simplified sensor array imaging setup

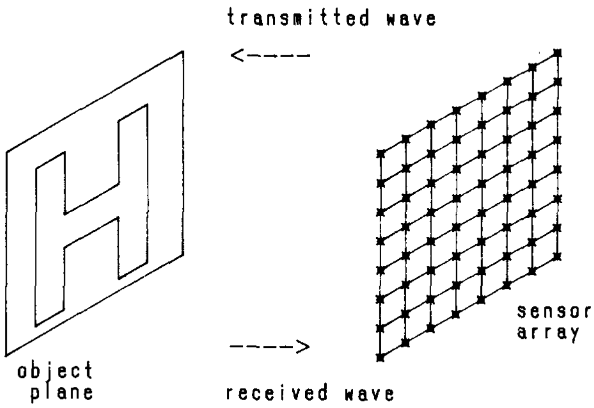

Fig. 1. A typical sensor array imaging system.

used in our studies. The field on the receiver array can be viewed as a digital hologram.

2. Need for simulation studies

Simulation of a sensor array imaging system involves determining the values of the complex field data at discrete points on the receiving plane. In order to compute these values, it is necessary to know the distribution of the wave field on the object plane. We make some approximations to determine this field distribution. These approximations will be closer to the true values only for simple cases of objects. We have chosen planar objects which are cut-outs of letters of the Roman script. Throughout these studies, by image reconstruction we mean the recovery of uniform regions in the original image.

The field distribution on the object plane is assumed to be zero outside the object region and a constant value on the object. This is similar to Kirchhoff's approximation in geometric optics and is valid when the wavelength is much smaller than the linear dimensions of the object.

Figure 2(a) shows the image used in our studies. The number of points on the object plane is 128×128. Figures 2(b) and 2(c) show the images formed from only the magnitude and only the phase information, respectively, of the received data at all the 128×128 points on the receiver plane. We see that the image formed from phase captures the essential features (edge information) of the original object. Thus even for imaging, as in image processing the phase information is more important than the magnitude information [5]. Later we will show that it is possible to form images even from quantized phase data. The reason for exploring the possibility of reconstruction from partial magnitude or phase of the received data is that the measurement complexity can be reduced significantly, by specifying only the significant bits of the received data.

Studies have been made by varying the number of points as well as the distribution of the sensor elements on the array. The effect of sparse data is illustrated in Fig. 3. The following cases are considered where the points on the receiver plane are assumed to be uniformly spaced:

(a) Receiver array containing 128×128 elements. Since the object also has 128×128 points, the measured data is sufficient for a

Fig. 2. Image reconstruction from magnitude/phase information of the received data. Original object: 128×128 points; number of receiver elements: 128×128. (a) Original image. (b) Reconstruction from magnitude information. (c) Reconstruction from phase information.

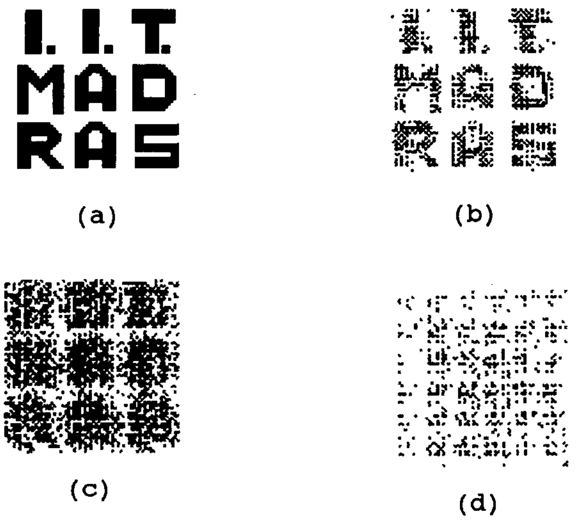

Fig. 3. Images reconstructed by varying the number of sensors on the receiver array. Original object: 128 × 128. (a) Number of receiver elements: 128 × 128. (b) Number of receiver elements: 64 × 64. (c) Number of receiver elements: 32 × 32. (d) Number of receiver elements: 16 × 16.

complete recovery of the original image (Fig. 3(a)).

(b) Receiver containing 64 × 64 elements (Fig. 3(b)).

(c) Receiver containing 32 × 32 elements (Fig. 3(c)).

(d) Receiver containing 16 × 16 elements (Fig. 3(d)).

It is seen from these figures that the clarity of the reconstructed image decreases with decreasing number of elements on the receiver array. If it is possible to obtain a good image of size, say 128 × 128 points, from data measured at a small number of points, say 32 × 32, then the size complexity of the receiver array can be reduced to practically realizable levels.

3. Algorithms for reconstruction from partial data

The problem of sensor array imaging can be viewed as signal reconstruction from partial data. The data in these cases are partial due to the finite number of receiver elements and also due to noise in the measurements. Recently [3, 8, 13] several iterative algorithms have been proposed for signal reconstruction from partial data, especially from magnitude or phase of the Fourier transform of the signal. We examine the applicability of these algorithms for sensor array imaging situations. The objective is to see if the complexity of receiver measurements can be reduced either by reducing the number of elements or by using quantized data or both. In particular we shall examine the reconstruction from Fourier transform magnitude alone, Fourier transform phase alone and quantized (1-bit and 2-bit) phase with magnitude information.

The Papoulis–Gerchberg algorithm [8] and the POCS (Projection Onto Convex Sets) technique [3, 13] are the most popular iterative algorithms for signal reconstruction from partial data. In this work, we have used the POCS algorithm for signal reconstruction. The general POCS algorithm is as follows:

/* Assume that $C_1, C_2, \ldots, C_n$ are the n convex sets formed from the available information. */

(1) Choose an initial estimate for the required solution.

repeat

(2) take projection onto set C_1.

(3) take projection onto set C_2.

⋮

$(n+1)$ take projection onto set C_n.

until satisfactory solution obtained

$(n+2)$ stop.

The available information can be used appropriately in the above algorithm to form specific algorithms according to the requirements.

The information we use for image formation in sensor array imaging is

(1) Phase of the received data.

(2) 2-bit phase of the received data.

(3) 1-bit phase of the received data.

(4) Finite support constraint in the image domain.

All these types of information define convex sets [14]. Therefore the POCS algorithms is well suited for our work.

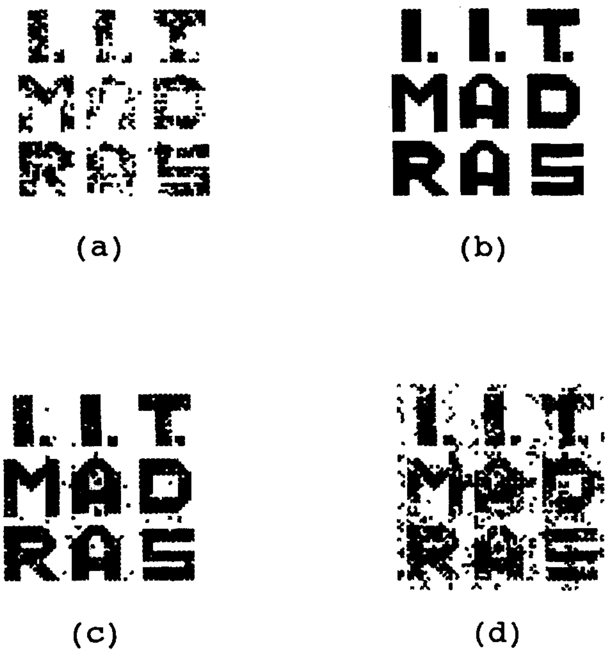

Fig. 4. Images reconstructed for different cases after 25 iterations. Original object: 128×128 points; number of receiver elements: 128×128. (a) Only magnitude. (b) Only phase. (c) 2-bit phase with constant magnitude. (d) 1-bit phase with constant magnitude.

4. Image formation from quantized phase data

Under certain situations, it is possible to recover a signal from only the phase of its Fourier transform [5]. We study the possibility of image formation from only the phase of the received data in sensor array imaging. Figures 4(a) and 4(b) show the images formed from only the magnitude and only the phase information respectively of the received data. The results are obtained after 25 iterations using finite support constraint in the image domain. We notice that it is possible to get the complete image information from full phase. Next we examine the possibility of reconstruction from quantized phase, as for the case of image processing [1]. Use of the quantized phase reduces the receiver complexity in the sense that it is sufficient to know one or two most significant bits of each phase value.

1-bit phase is the phase quantized to two levels according to the following scheme:

$$\theta_q = 0, \quad \text{for} \quad -\pi/2 < \theta < \pi/2,$$
$$= \pi, \quad \text{for} \quad \pi/2 < \theta < 3\pi/2.$$

2-bit phase is the phase quantized to four levels as follows:

$$\theta_q = \begin{cases} \pi/4, & \text{for} \quad 0 < \theta < \pi/2 \\ 3\pi/4, & \text{for} \quad \pi/2 < \theta < \pi \\ 5\pi/4, & \text{for} \quad \pi < \theta < 3\pi/2, \\ 7\pi/4, & \text{for} \quad 3\pi/2 < \theta < 2\pi. \end{cases}$$

Figures 4(c) and 4(d) show the image reconstructed from the quantized phase information of the received data. Figure 4(c) shows the image formed from 2-bit phase information alone. Figure 4(d) shows the image formed from only the 1-bit phase information of the received data. In both these cases, the initial estimate was formed from the correct quantized phase information and a flat magnitude spectrum.

5. Reduction in number of receiver elements

Phase quantization helps in reducing the circuit complexity in sensor array imaging. But the number of data samples required to reconstruct the signal from quantized phase information is generally more than in the case of signal reconstruction from full phase information. To collect the required number of data samples, the receiver array should have a large number of sensors. We had started with the aim of reducing the circuit complexity to make imaging through sensor array data easier. But the increased receiver array size more than offsets the reduction of circuit complexity obtained by using quantized phase information.

Earlier attempts to reduce the receiver array size used synthetic aperture techniques [6, 7, 9]. But these techniques do not make use of the advantages of iterative algorithms for signal reconstruction.

Some of the advantages are improvement of image quality and noise reduction. Our image formation technique is based on the POCS algorithm. In the POCS algorithm, convex sets are formed from the known information about the signal and the collected data. The algorithm converges to one of the elements in the intersection of these convex sets. If this intersection set is small, the solution obtained by using the POCS algorithm is close to the original signal. If this set is large, then the solution may be totally different from the original signal.

We propose a new technique for image reconstruction using data collected at several frequencies [11, 12]. The technique reduces the size of the intersection of various convex sets and then makes use of the POCS algorithm to obtain the solution. Assume that $f_0, f_1, \ldots, f_{N-1}$ are the N frequencies used for data collection. The data are collected by transmitting the wave at each frequency separately and then measuring the field induced at the receiver end due to each of them. These data can then be used in the following algorithm to reconstruct the image.

(1) Take the data collected at frequency f_0 as the starting point. Use these data to form the first estimate of the image of the object. Set a variable i to 0.

repeat

(2) Increment i by 1. Use the estimate of the image of the object formed at this stage to simulate the data at the receiver end for frequency $f_{i \bmod N}$.

(3) In accordance with the POCS algorithm, correct these simulated data with the actually known data samples at the frequency $f_{i \bmod N}$.

(4) Form the next estimate of the image of the object from these corrected data.

until an acceptable image is formed.

(5) Stop.

The above algorithm gives a general procedure to form an image from multiple frequency data. Any type of information that defines convex sets can be used for data correction in Step 3. In sensor array imaging, the received data are related to the field on the object plane by the following relation:

$$g_f(x, y) = g_0(x, y) * h_f(x, y), \tag{5}$$

where $g_0(x, y)$ is the field on the object plane and $h_f(x, y)$ is a factor that arises due to propagation of the wave from the object to the receiver array. This factor is a function of the frequency f.

For a discrete case it can be written as

$$g_f(m, n) = g_0(m, n) * h_f(m, n), \tag{6}$$

or

$$g_f(m, n) = \sum_{m'} \sum_{n'} g_0(m', n') h_f(m - m', n - n'). \tag{7}$$

The partial data collected at each frequency form a convex set. Each point in the set represents a solution to the constraints defined by the data collected at that frequency, and all possible solutions for the collected data are contained in this set. Signals that satisfy the equations for all frequencies lie in the intersection of the convex sets defined by the multiple frequency data. If the various convex sets have only one common point, it implies that the collected data define a unique solution. In that case, the algorithm given earlier will converge to it.

If the data are collected at only one frequency, the solution obtained by the POCS algorithm will be a member of the intersection of the convex sets formed from the collected data and the finite support constraint. If more data are collected, this time at a different frequency, the set of solutions for these data may be different from the solution set for the data at the first frequency. Therefore, the solution obtained from data collected at two frequencies will lie in the intersection of three convex sets: (1) the set formed from the finite support constraint, (2) the set formed from the data at first frequency, and (3) the set formed from the data at the second frequency. This solution set will be a subset of the previous solution set. Therefore, we expect the solution set to decrease in size

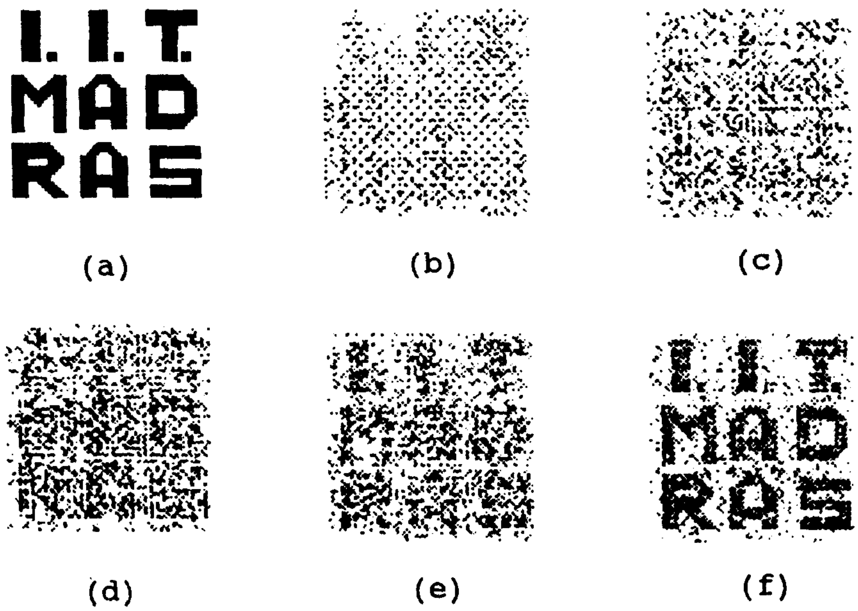

Fig. 5. Images reconstructed from full phase data using multiple frequencies by an array of 32 × 32 elements. Number of iterations: 50. (a) Original object. (b) Single frequency ($\lambda = 0.25$). (c) Two frequencies ($\lambda = 0.25, 0.26$). (d) Four frequencies ($\lambda = 0.25, 0.26, \ldots, 0.28$). (e) Eight frequencies ($\lambda = 0.25, 0.26, \ldots, 0.32$). (f) Sixteen frequencies ($\lambda = 0.25, 0.255, \ldots, 0.325$).

as we collect data at various frequencies. So the solution obtained by the POCS algorithm is expected to be closer to the original as compared with the solution obtained by data at a single frequency.

The number of frequencies required to make the measurements is dependent on the amount of known data at each frequency. If a small number of samples is known for each frequency, then the number of frequencies required for signal recovery will be large as compared with the situation when we have comparatively more information at each frequency.

Figs. 5(b)-(f) show the images formed from full phase information of the data collected by an array of 32 × 32 sensors using one ($\lambda = 0.25$), two ($\lambda =$ 0.25, 0.26), four ($\lambda = 0.25$, 0.26, 0.27, 0.28), eight ($\lambda = 0.25$, 0.26, ..., 0.32), and sixteen ($\lambda = 0.25$, 0.255, ..., 0.325) frequencies, respectively. The corresponding images formed from 2-bit phase information are shown in Fig. 6 and those formed from 1-bit information are shown in Fig. 7. In these cases, the magnitude information was used to form the initial estimate. All the results are after 50 iterations.

From the experimental results we note that a decrease in the amount of data because of the reduced number of sensors can be compensated by increasing the number of frequencies for reconstruction.

6. Noise reduction in sensor array imaging

The problem of noise in signals has been studied extensively [2, 4]. We can model the problem of noise in the following way. Let x be the wave field on the object plane. The propagation of the signal over the channel can be modelled as a transformation of the original signal. These transformed data are measured at the receiver end using an array of sensors. The received data y can be written as

$$y = Hx, \tag{8}$$

where H represents the transformation operator. In sensor array imaging, H is computable.

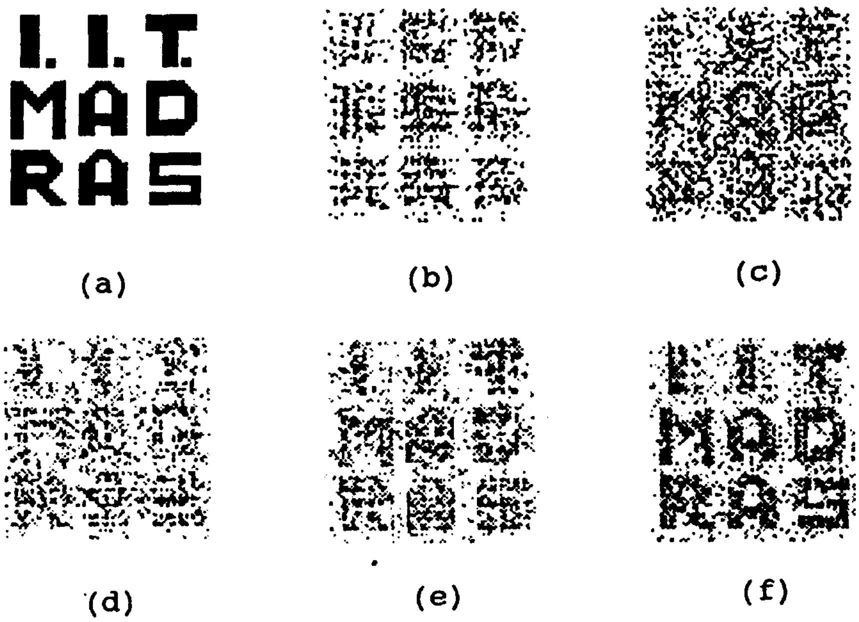

Fig. 6. Images reconstructed from 2-bit phase data using multiple frequencies by an array of 32 × 32 elements. Number of iterations: 50. (a) Original object. (b) Single frequency ($\lambda = 0.25$). (c) Two frequencies ($\lambda = 0.25, 0.26$). (d) Four frequencies ($\lambda = 0.25, 0.26, \ldots, 0.28$). (e) Eight frequencies ($\lambda = 0.25, 0.26, \ldots, 0.32$). (f) Sixteen frequencies ($\lambda = 0.25, 0.255, \ldots, 0.325$).

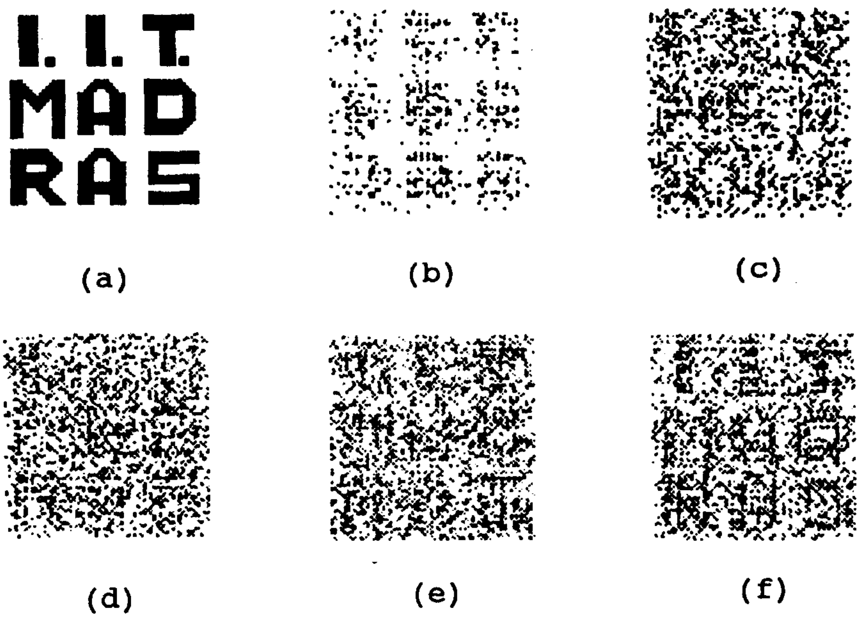

Fig. 7. Images reconstructed from 1-bit phase data using multiple frequencies by an array of 32 × 32 elements. Number of iterations: 50. (a) Original object. (b) Single frequency ($\lambda = 0.25$). (c) Two frequencies ($\lambda = 0.25, 0.26$). (d) Four frequencies ($\lambda = 0.25, 0.26, \ldots, 0.28$). (e) Eight frequencies ($\lambda = 0.25, 0.26, \ldots., 0.32$). (f) Sixteen frequencies ($\lambda = 0.25, 0.255, \ldots, 0.325$).

Therefore x can be recovered from y by using

$$x = H^{-1}y. \tag{9}$$

Equation (8) holds only in ideal situations. In the presence of noise, the received data can be written as

$$y = Hx + n, \tag{10}$$

where n is the channel and circuit noise. To recover the original signal, the noise must be removed.

A number of image processing techniques have been developed for noise cleaning in the image domain. The image formed in sensor array imaging has poor resolution. Therefore if the effects of noise are not reduced during image reconstruction, the image formed may be so bad that the image processing techniques may not be useful.

Keeping this in mind, we have tried to address the problem of noise reduction during the image reconstruction stage itself. Addition of noise changes both magnitude and phase values in the received data. If full phase is used in the POCS algorithm, every data point is forced to have an incorrect phase value. On the other hand, additive noise may not affect most of the quantized phase values. So the image formed from the quantized phase information of the received data is expected to be better than that formed from the full phase information, when the noise level is sufficiently low.

The main idea in using the quantized phase information for image formation in the presence of noise is to use a small amount of right information rather than a large amount of incorrect information. Even if the full phase measurement is available, it is better to use quantized phase information for checking during iterations. The full phase information of the data can be used to form an initial estimate of the image.

The use of quantized phase information for noise reduction is more evident when the receiver array contains a small number of sensors, and multiple frequency data are used for image formation. In this case we can choose data points where we are confident of the correctness of the sign information. For this we may choose samples with high magnitude values because of the lessened effect of noise at these points and therefore the quantized phase information at these points is expected to be less affected by noise. The data are now sparse because of the reduced sampling. Therefore more than one solution may be possible for the collected data. It has been experimentally observed that the solution obtained from quantized phase of the received data reproduces the essential features of the original object, whereas the solution obtained from full noisy phase information does not reproduce these features. Since recognition of the features is important for the recognition of an object, the use of quantized phase information can be useful for image formation from noisy data.

Figure 8 shows the images formed from full phase, 2-bit phase and 1-bit phase information, for different levels of noise in the data collected by an array of 64×64 sensors at eight frequencies ($\lambda = 0.25, 0.26, \ldots, 0.32$ units). The images were obtained after 25 iterations of the POCS algorithm. For SNR = 12 dB, the image formed from the full phase information (Fig. 8(a)) is comparable with that formed from the 2-bit phase information (Fig. 8(b)). When SNR is reduced (−1.88 dB, and −20 dB cases), the images reconstructed from the 2-bit phase data (Figs. 8(e) and 8(h)), are better than those formed from the full phase information (Figs. 8(d) and 8(g)) or from the 1-bit phase data (Figs. 8(f) and 8(i)). But as the noise level is further increased, it is not possible to obtain images even from the 2-bit phase data. This is illustrated in Fig. 8(k). The images shown here were formed for an SNR of −30 dB. These studies show that for noise levels in some intermediate range, images formed from the 2-bit phase data are better than those formed from the full phase data.

One possible explanation for better performance of the quantized phase case compared with the full phase case is that in the latter case the noisy phase is substituted as known data in each iteration. On the other hand, in the quantized phase case, in each iteration only the quantized phase of the known noisy data is compared with the quantized

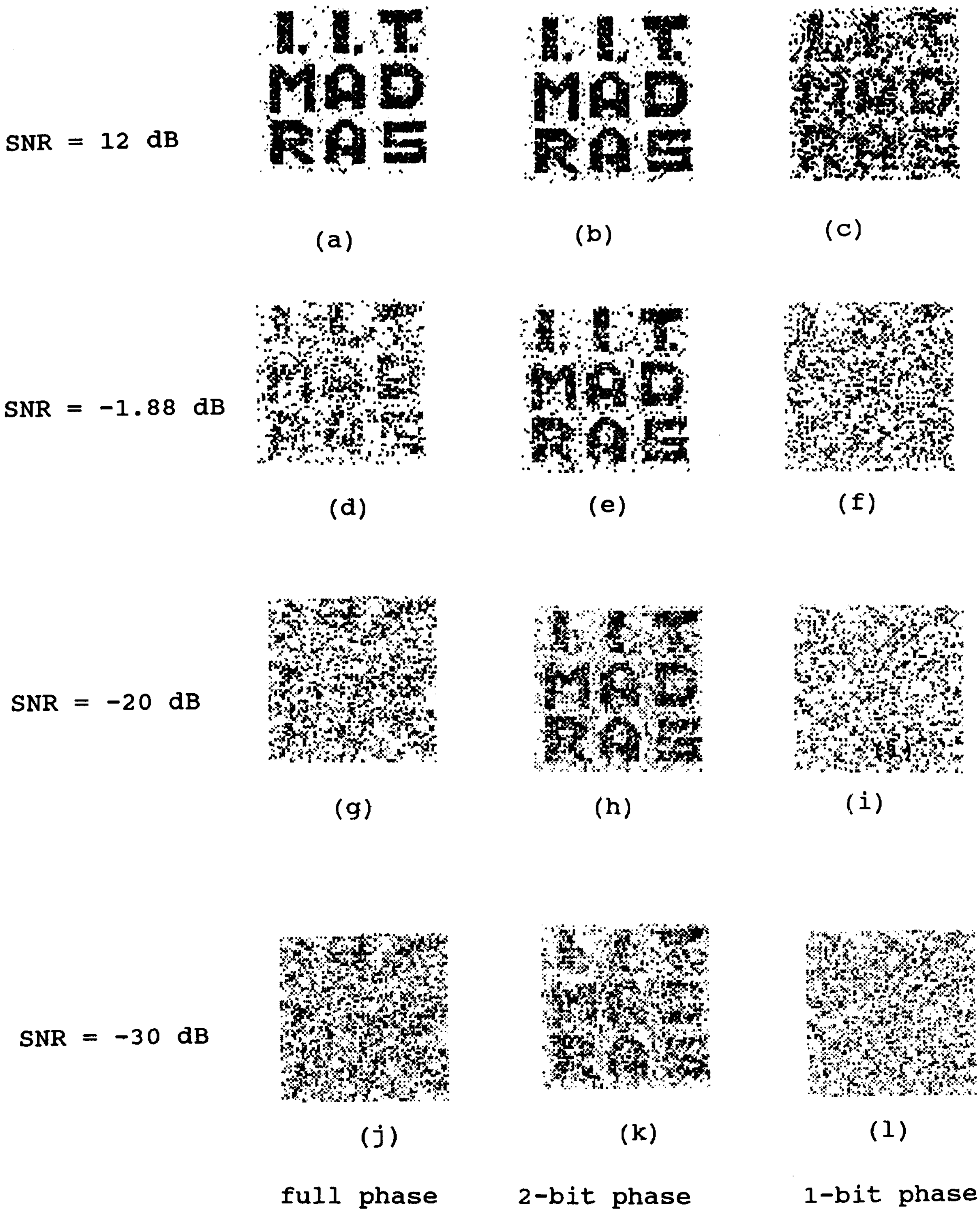

Fig. 8. Images reconstructed from full phase, 2-bit phase, 1-bit phase for different SNRs. Original object: 128 × 128 points. Number of iterations: 25. Number of receiver elements: 64 × 64. Number of frequencies: 8 (λ = 0.25, 0.26, ..., 0.28).

phase of the reconstructed phase. It is likely that the quantized versions may match at a greater number of sampling points as the number of iterations are increased.

7. Conclusions

In this paper we have discussed some signal processing issues that arose in sensor array imaging. We have used a simplified model of the imaging situation to focus on the problem of receiver complexity. Through simulation studies we have demonstrated that it is possible to reduce the measurement complexity by using only the quantized phase values. The receiver complexity can be reduced significantly by using only a small number of sensor elements if measurements are made at multiple frequencies. We have been able to accomplish the reduction in receiver complexity by increasing the computational complexity. With a small number of receiver elements a large number of iterations of the multiple frequency data are required to obtain an acceptable image quality. Thus it appears that the receiver complexity (number of sensor elements) can be reduced by increasing the computational complexity (number of iterations).

Acknowledgment

We thank the reviewers for their comments on the original version of the paper which enabled us to improve the presentation significantly.

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