intrinsic-component expansion with the least-squares truncation property. The interpolation of curves was reduced to the simple interpolation of weighting coefficients. The technique presented here, therefore, has some advantages from the data processing standpoint, which are data compression, reduction in processing time, and stability of the solution. In the application to the estimation problem of γ-ray response spectra, we have shown that the spectral curves can be represented by utilizing only two principal-component patterns. Moreover, we have investigated the accuracy of the estimated curves and constructed a library of the curves.

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Design of ARMA Digital Filters by Pole-Zero Decomposition

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Abstract—A new technique for design of digital filters is presented in this paper. The technique exploits the spectral approximation property of autoregressive modeling to reduce ripple at the edges of the transition band in the filter response. An autoregressive model approximates a given spectrum better at the peaks than at the valleys. Spectral information around the transition band is transformed into peaks by splitting the given squared-magnitude frequency response into two component spectra. This splitting is accomplished using a pole-zero decomposition technique, which in turn uses the properties of the derivative of phase spectrum of minimum phase filters. One of the component spectra corresponds nearly to the response of an all-pole filter, and the other component spectrum corresponds nearly to the response of an all-zero filter. Each of these component spectra can be represented by a small number of coefficients using autoregressive modeling. The resulting two sets of autoregressive coefficients determine poles and zeros of the autoregressive moving average (ARMA) digital filter. Ripple characteristics in the response of the ARMA filter can be controlled by appropriately choosing the number of poles and zeros. It is shown that a wide variety of magnitude frequency response characteristics can be approximated by an ARMA filter of low order using this technique. The technique does not work well in cases where spectral approximation by autoregressive modeling is poor. Such cases arise when the component spectra have very large dynamic range.

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I. INTRODUCTION

A DIGITAL filter that contains poles and zeros is termed an autoregressive and moving average (ARMA) digital filter. This paper is concerned with the design of ARMA digital filters to realize a given log magnitude frequency response. The basic idea in the design is to split the given response into two component responses, each of which can be

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approximated by a small number of parameters. The reason for the success of this method is that one of the component responses is close to an all-pole spectrum and the other component response is close to an all-zero spectrum. Since the inverse of an all-zero spectrum is an all-pole spectrum, it is possible to represent each of the component responses by a small number of parameters through autoregressive modeling [1]. The splitting of the given response into an all-pole and an all-zero spectra is accomplished using a pole-zero decomposition technique [2], which is based on the properties of the negative derivative of minimum phase spectra [3].

In a closely related study, Scharf and Luby suggested statistical design of ARMA digital filters [4]. The design procedure consists of determining, first, a higher order all-pole filter approximation for the given spectrum. The all-pole filter is used to generate consistent unit impulse and covariance sequences for use in the Mullis–Roberts algorithm [5]. This algorithm is then used to obtain a low-order ARMA digital filter that approximates the higher order all-pole filter.

Although the goal in our study is also to design a digital filter with poles and zeros, our method of approximation to obtain the filter coefficients is different. In the Scharf and Luby design, the ripple in the realized response is maximum at the edges of the transition band. Attempts to control this effect with smooth functional transition from passband to stopband have been unsuccessful. This is because the moving average (all-zero) whitening filter for the given spectrum tries to whiten a spectrum with zero amplitude in the transition band. The corresponding autoregressive (all-pole) filter has excessive ripple in the passband. This cannot be avoided because an autoregressive model approximates peaks in a spectrum better than valleys [1]. In the method proposed in this paper, this property of autoregressive modeling is exploited to effectively control the ripple at the band edges.

It is generally true that a given magnitude spectrum can be realized by a digital filter of a much lower order when the filter contains both poles and zeros than when the filter is purely all-pole or all-zero. A lower order digital filter will be of lesser complexity in terms of number of multiplications and additions required in its implementation. Our method of design results in an ARMA digital filter that is of low order and stable. In addition, the filter has several useful characteristics. The ripple characteristics can be controlled by a suitable choice of the number of coefficients used to represent the component spectra.

The emphasis in this paper is on the presentation of the new design technique. Issues such as the filter performance relative to other techniques and the limitations of the method are not discussed in detail. Throughout the paper the notation (M_1, M_2) denotes an ARMA digital filter with M_1 poles and M_2 zeros.

II. Theory of Pole-Zero Decomposition

The key idea in this paper is splitting the given log magnitude frequency response into two parts: one corresponding nearly to an all-pole filter spectrum and the other to an all-zero filter spectrum. This splitting is called pole-zero decomposition, which is based on the properties of the negative derivative of phase spectra (NDPS) of minimum phase polynomials.

A. *Properties of Minimum Phase Filters*

A linear digital filter $H(z)$ can be represented as a ratio of two polynomials as follows:

$$H(z) = GN(z)/D(z) \tag{1}$$

where

$$N(z) = 1 + \sum_{k=1}^{M_2} a^-(k)\, z^{-k} \tag{2}$$

$$D(z) = 1 + \sum_{k=1}^{M_1} a^+(k)\, z^{-k} \tag{3}$$

and G is a gain term.

A polynomial is said to be minimum phase if all its roots lie within the unit circle in the z-plane. If all the poles and zeros of $H(z)$ lie within the unit circle in the z-plane, then the filter is called a minimum phase filter. Properties of minimum phase signals have been studied extensively [6]. Properties of the negative derivative of phase spectrum (NDPS) of a minimum phase polynomial have been reported [3] in the context of formant extraction using linear prediction analysis.

The NDPS of a minimum phase polynomial

$$A(z) = 1 + \sum_{k=1}^{M} a(k)\, z^{-k} \tag{4}$$

is defined as follows.

Let

$$\tilde{A}(\omega) = A(z)\big|_{z=e^{j\omega}} \tag{5}$$

and

$$\tilde{A}(\omega) = |\tilde{A}(\omega)|\, e^{-j\theta_A(\omega)}. \tag{6}$$

Then the NDPS of $A(z)$ is given by $d\theta_A(\omega)/d\omega$.

A polynomial of the type $A(z)$ can be written as a cascade of several first-order polynomials with real roots and second-order polynomials with complex conjugate roots. The NDPS plots of typical first-order and second order all-pole filters are shown in Fig. 1. It is interesting to note that significant values of NDPS for a first-order real pole filter lie close to the origin of the frequency scale. For a second-order all-pole filter (resonator) the significant values of NDPS are confined to frequencies around the resonance frequency. Moreover, the NDPS curve near resonance frequency is approximately proportional to the squared-magnitude response of the filter. These properties were shown analytically in [3].

The NDPS of the overall all-pole filter $1/A(z)$ is a summation of the NDPS responses of the individual first and second order filters. In the overall response there is negligible effect of one resonance peak on the other as shown in Fig. 2.

It is easy to visualize a similar behavior for real and complex conjugate pair zeros in their NDPS plots. The only difference is that the NDPS for zeros will have a sign opposite to that for poles. Specifically, the NDPS plot will have a positive peak

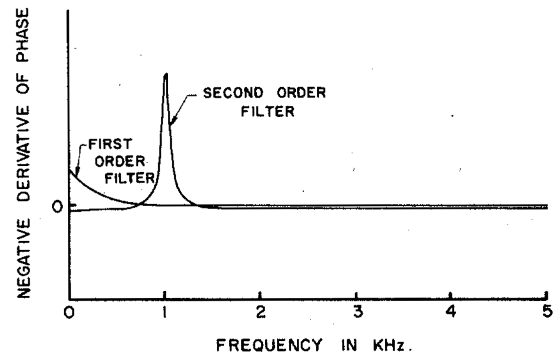

Fig. 1. Negative derivative of phase spectra for all-pole filters. First-order filter: $H(z) = 1/(1 - 0.85z^{-1})$. Second-order filter: $H(z) = 1/(1 - 1.57z^{-1} + 0.94z^{-2})$.

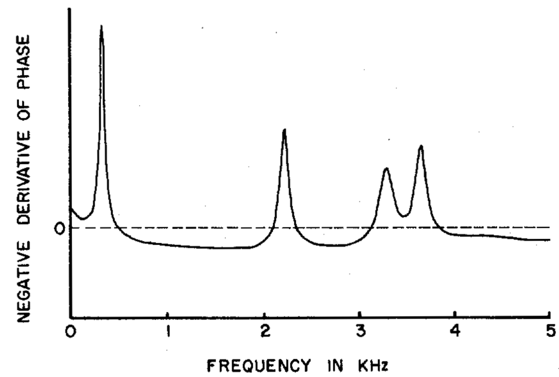

Fig. 2. Negative derivative of phase spectrum for cascade of several first-order and second-order all-pole filters.

due to a complex conjugate pole pair and a negative peak due to a complex conjugate zero pair. These simple but powerful properties of the derivative of phase spectrum are shown to accomplish the pole-zero decomposition discussed in Section II-C.

B. Relation Between Magnitude Spectrum and NDPS

Let $V(\omega)$ represent the frequency response of a minimum phase digital filter. $V(\omega)$ will be periodic in ω with period 2π. Since all the poles and zeros of the minimum phase filter lie within the unit circle in the z-plane, $\ln V(\omega)$ can be expressed in Fourier series expansion as in (7) below, since the cepstrum of a minimum phase sequence is a causal sequence [6].

$$\ln V(\omega) = c(0)/2 + \sum_{k=1}^{\infty} c(k)\, e^{-jk\omega} \tag{7}$$

where $\{c(k)\}$ are called cepstral coefficients. Writing

$$V(\omega) = |V(\omega)|\, e^{-j\theta_V(\omega)} \tag{8}$$

we get the real and imaginary parts of $\ln V(\omega)$ as

$$\ln |V(\omega)| = c(0)/2 + \sum_{k=1}^{\infty} c(k) \cos k\omega \quad \text{(real part)} \tag{9}$$

and

$$\theta_V(\omega) + 2\lambda\pi = \sum_{k=1}^{\infty} c(k) \sin k\omega \quad \text{(imaginary part)} \tag{10}$$

where λ is an integer. Taking the derivative of $\theta_V(\omega)$, we get

$$\theta_V'(\omega) = \sum_{k=1}^{\infty} k\, c(k) \cos k\omega. \tag{11}$$

$\theta_V'(\omega)$ represents the negative derivative of phase spectrum (NDPS) of the minimum phase filter. The log magnitude spectrum and the NDPS of a minimum phase filter are thus related through the cepstral coefficients.

C. Pole-Zero Decomposition

Separating the positive and negative parts of $\theta_V'(\omega)$, we get the approximate NDPS of the pole and zero components of the filter, respectively.

$$\theta_V'(\omega) = [\theta_V'(\omega)]^+ + [\theta_V'(\omega)]^- \tag{12}$$

where

$$[\theta_V'(\omega)]^+ = \theta_V'(\omega), \quad \text{for } \theta_V'(\omega) \geq 0 \quad \text{(NDPS of pole part)}$$
$$= 0, \quad \text{for } \theta_V'(\omega) < 0 \tag{13}$$

and

$$[\theta_V'(\omega)]^- = \theta_V'(\omega), \quad \text{for } \theta_V'(\omega) < 0 \quad \text{(NDPS of zero part)}$$
$$= 0, \quad \text{for } \theta_V'(\omega) \geq 0. \tag{14}$$

Expressing each of the NDPS responses separately in Fourier series yields the cepstral coefficients $\{c^+(k)\}$ and $\{c^-(k)\}$ corresponding to the pole and zero spectrum, respectively. That is

$$[\theta_V'(\omega)]^+ = C + \sum_{k=1}^{\infty} k\, c^+(k) \cos k\omega \tag{15}$$

and

$$[\theta_V'(\omega)]^- = -C + \sum_{k=1}^{\infty} k\, c^-(k) \cos k\omega \tag{16}$$

where C is the average value, which does not contribute to the shape of the spectrum. From $\{c^+(k)\}$ and $\{c^-(k)\}$ the pole spectrum and zero spectrum can be computed through Fourier cosine transform and exponentiation.

III. Theory of ARMA Filter Design

A. Filter Design Problem

Given a squared-magnitude frequency response $S(\omega)$, the objective in filter design is to determine the parameters of a linear digital filter $H(z)$ of the type given in (1), such that

$$|\tilde{H}(\omega)|^2 = |H(e^{j\omega})|^2$$
$$\approx S(\omega). \tag{17}$$

B. Design Procedure

The theory of the ARMA filter design is illustrated through the following design steps. The actual implementation of the design is discussed in Section IV.

1) For the given squared-magnitude frequency response $S(\omega)$ determine the cepstral coefficients $\{c(k)\}$ using the

relation

$$\ln S(\omega) = c(0) + 2 \sum_{k=1}^{\infty} c(k) \cos k\omega. \tag{18}$$

2) From $\{c(k)\}$ compute the NDPS $\theta'_V(\omega)$ for the minimum phase filter corresponding to $S(\omega)$, using the relation (11).

3) Split the NDPS $\theta'_V(\omega)$ into positive and negative portions using the relations (13) and (14).

4) Obtain the cepstral coefficients $\{c^+(k)\}$ and $\{c^-(k)\}$from $[\theta'_V(\omega)]^+$ and $[\theta'_V(\omega)]^-$, respectively, using the relations (15) and (16).

5) Compute the pole spectrum $P(\omega)$ and the zero spectrum $Z(\omega)$ from $\{c^+(k)\}$ and $\{c^-(k)\}$, respectively, as follows:

$$P(\omega) = \exp\left[c(0)/2 + 2 \sum_{k=1}^{\infty} c^+(k) \cos k\omega\right] \tag{19}$$

and

$$Z(\omega) = \exp\left[c(0)/2 + 2 \sum_{k=1}^{\infty} c^-(k) \cos k\omega\right]. \tag{20}$$

Note that $c^+(k) + c^-(k) = c(k)$ and, hence, $P(\omega)\, Z(\omega) = S(\omega)$.

6) Obtain the autocorrelation coefficients $\{R^+(k)\}$ and $\{R^-(k)\}$ from $P(\omega)$ and $1/Z(\omega)$ using the relations

$$P(\omega) = R^+(0) + 2 \sum_{k=1}^{\infty} R^+(k) \cos k\omega \tag{21}$$

and

$$1/Z(\omega) = R^-(0) + 2 \sum_{k=1}^{\infty} R^-(k) \cos k\omega. \tag{22}$$

7) Solve for the autoregressive coefficients $\{a^+(k)\}$ and $\{a^-(k)\}$ from $\{R^+(k)\}$ and $\{R^-(k)\}$, respectively, using Durbin's algorithm for solving the autocorrelation normal equation [1]. The equations are

$$\sum_{k=1}^{M_1} a^+(k) R^+(|i-k|) = -R^+(i), \quad i = 1, 2, \cdots, M_1, \tag{23}$$

and

$$\sum_{k=1}^{M_2} a^-(k) R^-(|i-k|) = -R^-(i), \quad i = 1, 2, \cdots, M_2. \tag{24}$$

8) Compute the filter gain G as

$$G = \exp\,[c(0)/2]. \tag{25}$$

9) The squared-magnitude frequency response of the overall filter is given by

$$|\tilde{H}(\omega)|^2 = \hat{P}(\omega)\, \hat{Z}(\omega)\, G^2 \tag{26}$$

where $\hat{P}(\omega)$ and $\hat{Z}(\omega)$ are approximations to $P(\omega)$ and $Z(\omega)$, respectively, and are given by

$$\hat{P}(\omega) = 1 \Bigg/ \left|1 + \sum_{k=1}^{M_1} a^+(k) e^{-jk\omega}\right|^2 \tag{27}$$

and

$$\hat{Z}(\omega) = \left|1 + \sum_{k=1}^{M_2} a^-(k) e^{-jk\omega}\right|^2. \tag{28}$$

IV. Implementation of the ARMA Filter Design

The design procedure described in Section III can be implemented as shown in Fig. 3. The desired filter response is specified by giving the log spectral values at sample points in the frequency domain. The input and output data for each DFT and IDFT block is real and symmetric. Small values of the imaginary part, that may occur as a result of finite precision arithmetic in FFT computation, are set to zero. The design examples discussed in the next section were implemented on a DEC KL10 machine (word length 36 bits).

V. Design Examples

A low-pass filter with the following specifications is considered for illustrating the above design procedure.

Let

$$Q(l) = S(\omega)\big|_{\omega = 2\pi l/512},$$

A = amplitude and M = number of transition samples.

A sampling frequency of 10 kHz is arbitrarily assumed in the discussion of design examples. In the examples 512 samples correspond to 10 kHz.

Specifications:

$$\begin{aligned}\ln Q(l) &= \ln(A), \quad l = 0, 1, \cdots, 99\\ &= [1 - (l-99)/(M+1)] \ln(A),\\ &\qquad l = 100, 101, \cdots, 100+M\\ &= 0, \quad l = 100+M+1,\ 100+M+2, \cdots, 256\\ \ln Q(l) &= \ln Q(512-l), \quad l = 257, 258, \cdots, 511.\end{aligned}$$

In the above specification the value of A determines the passband-to-stopband level. For example, if $A = 10^6$, then the passband-to-stopband level is 60 dB. The value of M determines the number of transition samples. The case of $M = 0$ corresponds to no sample in the transition band. The filter design was carried out using 512 point FFT for computing DFT and IDFT.

Fig. 4 illustrates the principle of the proposed filter design. Fig. 4(a) shows the log magnitude response of the desired low-pass filter for $A = 10^6$, $M = 21$. The NDPS of the filter is shown in Fig. 4(b). The positive and negative portions of the NDPS are separated and the corresponding spectra are computed. The resulting pole and zero spectra are shown in Fig. 4(c). If these log spectra are added, we get the desired filter response exactly as shown in Fig. 4(d). On the other hand, if the pole spectrum and the inverse of the zero spectrum are approximated by autoregressive models, each with 8 coefficients, the overall response of the resulting filter is as shown in Fig. 4(e). In this case, the peak-to-peak amplitude of the ripple is less than 3 percent in passband and stopband. An interesting observation is that the amplitude of the ripple depends mainly on the rate of fall during transition rather than the transition width itself. This

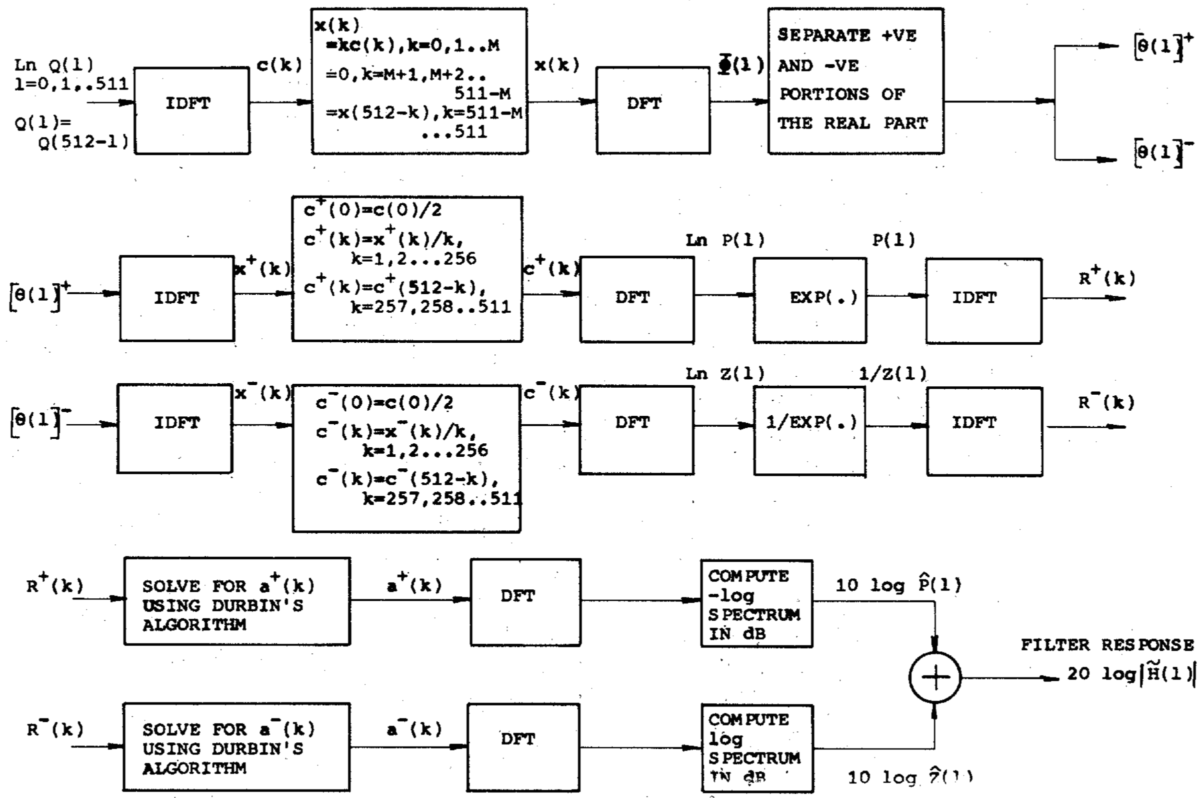

Fig. 3. Block diagram showing implementation details of the ARMA filter design.

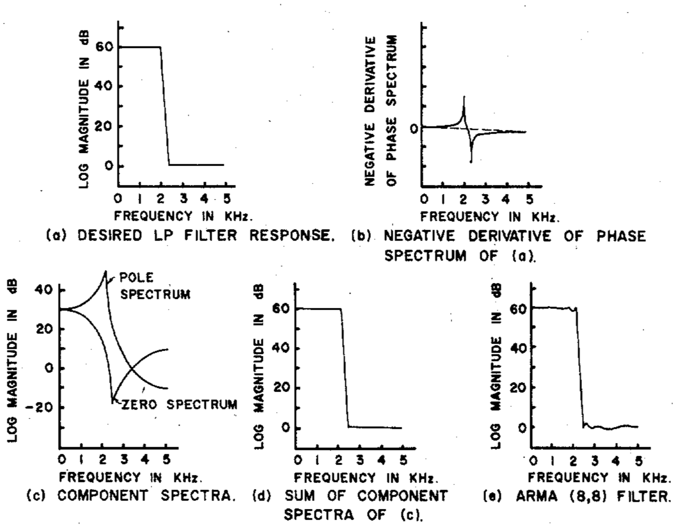

Fig. 4. Design of low-pass filter with a transition width of 21 samples (256 samples correspond to 5 kHz).

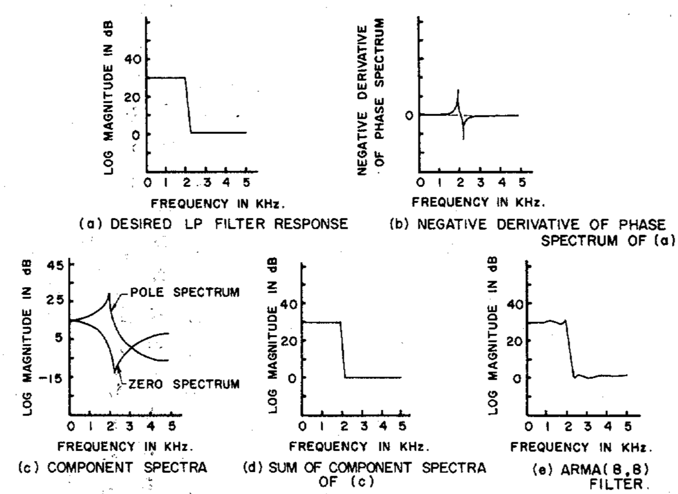

Fig. 5. Design of low-pass filter with a transition width of 11 samples (256 samples correspond to 5 kHz).

is illustrated in Fig. 5, where the different curves of Fig. 4 are obtained for $A = 10^3$, $M = 11$. The ripple amplitude in this case is less than 6 percent.

That the amplitude of the ripple can be traded with either the width of the transition band or the complexity (order) of the filter is illustrated in Fig. 6 for $A = 10^3$. The filter responses for four different orders of the filter and three different transition widths are given in the figures. For a (16, 16) filter, for example, the ripple amplitude reduces from 11 percent for $M = 1$ to 4 percent for $M = 11$. This tradeoff characteristic of the ripple amplitude with transition width makes this design superior to the statistical design of ARMA digital filters reported in [4].

The effect of varying the number of zeros keeping the number of poles fixed is shown in Fig. 7 for $A = 10^6$, $M = 21$. The passband characteristics are not significantly affected by changing the number of zero coefficients except when the number is too small. Similarly, we observed that the stopband characteristics are not significantly affected by changing the number of pole coefficients. This will provide the flexibility to design a filter with desired passband and stopband characteristics.

The number of points (N) for FFT computation depends upon the frequency sampling rate used in the specification.

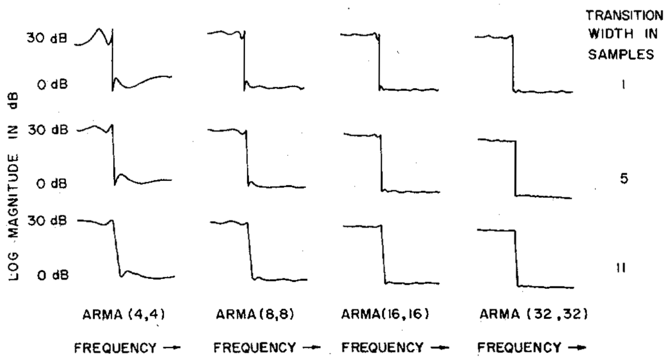

Fig. 6. Tradeoff between ripple amplitude and width of the transition band for different orders of the filter.

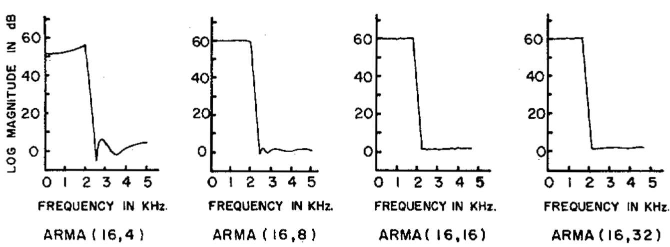

Fig. 7. Effect of varying the number of zeros of ARMA digital filter (transition width = 21 samples, 256 samples correspond to 5 kHz).

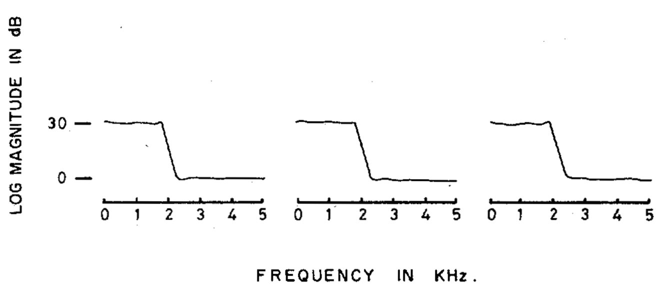

Fig. 8. Effect of frequency sampling rate on the response of an ARMA (8, 8) digital filter. (a) 128 samples/10 kHz. b) 256 samples/10 kHz. (c) 512 samples/10 kHz. The transition width in Hz is same in all cases, but in samples it is 5 in (a), 10 in (b), and 20 in (c).

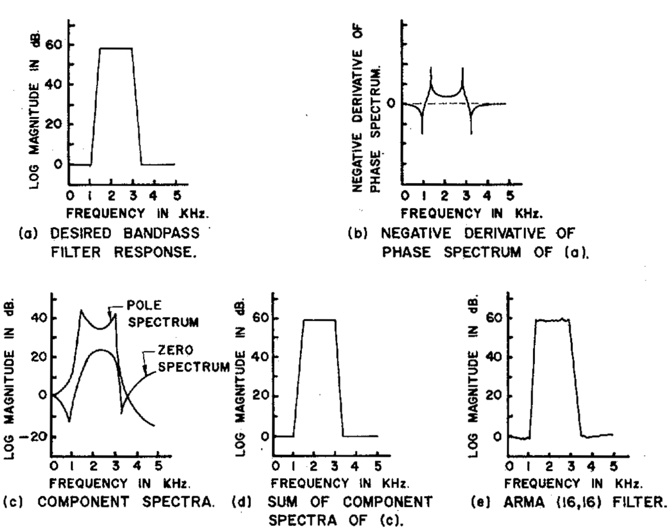

Fig. 9. Design of bandpass filter with a transition width of 21 samples (256 samples correspond to 5 kHz).

The effect of different sampling rates for a fixed transition width in Hz is shown in Fig. 8. The number of samples in the transition width are $M = 4$, 9, and 19 for $N = 128$, 256, and 512, respectively. The corresponding frequency sampling rates are 128 samples/10 kHz, 256 samples/10 kHz, and 512 samples/10 kHz, respectively. It is clear that there is very little effect of sampling rate on the realized response, at least for the transition width and sampling rates shown in the figure.

Finally, the design of a bandpass filter is illustrated in Fig. 9. This shows that any arbitrary filter characteristics can be realized using the technique presented in this paper.

VI. Concluding Remarks

We have shown that the pole-zero decomposition technique provides an effective method for designing ARMA digital filters. The complexity of the filter can be traded with the width of the transition band. By varying the number of poles and zeros independently, a wide variety of filter characteristics can be achieved. Since the roots of the resulting numerator and denominator polynomials lie within the unit circle in the z-plane, the filter and its inverse both are guaranteed to be stable. Therefore, if $H(z)$ represents a low-pass filter, then $1/H(z)$ represents a high-pass filter. Similar reasoning applies for bandpass and band elimination filters also.

The method is useful as a filter design procedure because it reduces the ripple at the bandedge by exploiting the property of spectral approximation by autoregressive modeling. An autoregressive model approximates a spectrum better at the peaks than at the valleys. The bandedge information is transformed into peaks in the component spectra using the pole-zero decomposition technique. Since both peaks and valleys of the given spectrum are represented equally well by the model, the method can be used to realize an arbitrary magnitude response by an ARMA filter of low order.

Computation of the component spectra $P(\omega)$ and $Z(\omega)$ through exponentiation as in (19) and (20) assures that $P(\omega)$ and $Z(\omega)$ are positive spectra (i.e., $P(\omega) > 0, Z(\omega) > 0$. This, in turn, assures that the autocorrelation coefficients $\{R^+(k)\}$ and $\{R^-(k)\}$, obtained through inverse Fourier transform of $P(\omega)$ and $1/Z(\omega)$, respectively, are positive definite [7]. Hence, the solutions of the normal equations (23) and (24) give coefficients $\{a^+(k)\}$ and $\{a^-(k)\}$, which guarantee that all the roots of $D(z)$ and $N(z)$ in (1) lie inside the unit circle in z-plane [1]. This results in stable $H(z)$ and $1/H(z)$. The positive definiteness of the autocorrelation coefficients can be lost if one uses small word lengths to represent the coefficients in a computer. Also, the round-off errors can cause the autocorrelation matrix in normal equations to become ill conditioned [1]. Therefore, it is often necessary to check for the stability of the filter, especially while using a small word length machine and/or while designing a high-order ARMA digital filter.

Since the design mainly involves computations using FFT algorithms, it will be convenient to implement it on a dedicated signal processing system consisting of Fourier analysis. In fact, the design was implemented successfully on an HP 5451-B Fourier Analyzer System, which uses a 16-bit HP 2100S minicomputer as the main processor. The total computation time for the design of an ARMA filter is slightly more than the computation time for eight DFT operations.

The number of DFT computations can be reduced to six if we use the relation $S(\omega) = P(\omega)\,Z(\omega)$. In such a case, once $P(\omega)$ is computed from $[\theta_V'(\omega)]^+$, $Z(\omega)$ can be obtained as $S(\omega)/P(\omega)$, instead of computing it through $[\theta_V'(\omega)]^-$. [1]

Our method is based on the spectral approximation property of autoregressive modeling, the accuracy of which depends on the spectral dynamic range. The dynamic ranges of the component spectra depend on the width of the transition band. The component spectral dynamic range is larger for smaller widths. Spectral approximation by autoregressive modeling of large dynamic range spectra will be generally poor, especially at low values of the spectrum. This, together with the finite word length computation, limit the type of ARMA filters that can be designed using our method.

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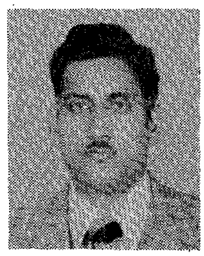
[1]The author would like to thank the reviewer for suggesting this idea for reducing computation.



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Tracking Properties of Adaptive Signal Processing Algorithms

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Abstract—**Adaptive signal processing algorithms are often used in order to "track" an unknown time-varying parameter vector. This work develops an upper bound on the mean of the norm-squared error between the unknown parameter vector being tracked and the value obtained by the algorithm. The results require very mild covariance decay rate conditions on the training data and a bounded algorithm. The upper bound illustrates the relationship between the algorithm step size and the maximum rate of variation in the parameter vector being tracked.**

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I. INTRODUCTION

MANY of the algorithms proposed for use in adaptive signal processing (see, e.g., [1]–[6]) can be cast in the following form:

$$W_{k+1} = W_k + \mu_k(P_k - F_k W_k), \quad k = 1, 2, \cdots \tag{1}$$

where $W_1 \in \Re^p$ is arbitrary, $\{P_k\}$ and $\{F_k\}$ are sequences of real $p \times 1$ and $p \times p$ matrices, respectively, and $\{\mu_k\}$ is a sequence of positive constants. We use $\Re^p$ to denote the real Euclidean p-space and treat $X \in \Re^p$ as a column vector. Algorithm (1) can be interpreted as a general member of a

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